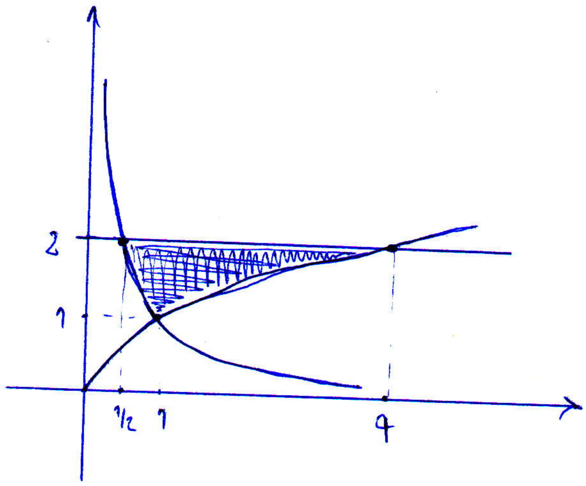
1. Izračunajte

$$\int\int_S 2y \; dxdy,$$

gdje je S područje omeđeno krivuljama $y = \frac{1}{x}$, $y = \sqrt{x}$ i $y = 2$. (3 boda)

$x = \frac{1}{y}$, $x = y^2$



1. način

$$\int_1^2 dy \int_{1/y}^{y^2} 2y \, dx = \int_1^2 dy \; 2yx \Big|_{1/y}^{y^2} = \int_1^2 dy \left(2y^3 - 2\right) =$$

$$= \left(2\,\frac{y^4}{4} - 2y\right)\Big|_1^2 = 8 - 4 - \left(\frac{1}{2} - 2\right) = \frac{11}{2}$$

2. način

$$\int_{1/2}^{1} dx \int_{1/x}^{2} 2y \, dy + \int_1^4 dx \int_{\sqrt{x}}^{2} 2y \, dy = \int_{1/2}^{1} dx \; 2\,\frac{y^2}{2}\Big|_{1/x}^{2} + \int_1^4 dx \; 2\,\frac{y^2}{2}\Big|_{\sqrt{x}}^{2} =$$

$$= \int_{1/2}^{1} \left(4 - \frac{1}{x^2}\right) dx + \int_1^4 (4 - x) \, dx = \left(4x + \frac{1}{x}\right)\Big|_{1/2}^{1} + \left(4x - \frac{x^2}{2}\right)\Big|_1^4 =$$

$$= 4 + 1 - (2 + 2) + 16 - 8 - \left(4 - \frac{1}{2}\right) = 5 + \frac{1}{2} = \frac{11}{2}$$

2. (i) Skicirajte površinu određenu integralom

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\phi \int_{\frac{4}{\sin\phi+\cos\phi}}^{\frac{4}{\sin\phi}} r dr.$$

(1 bod)

$$r = \frac{4}{\sin\varphi + \cos\varphi}$$

$$r\sin\varphi + r\cos\varphi = 4$$

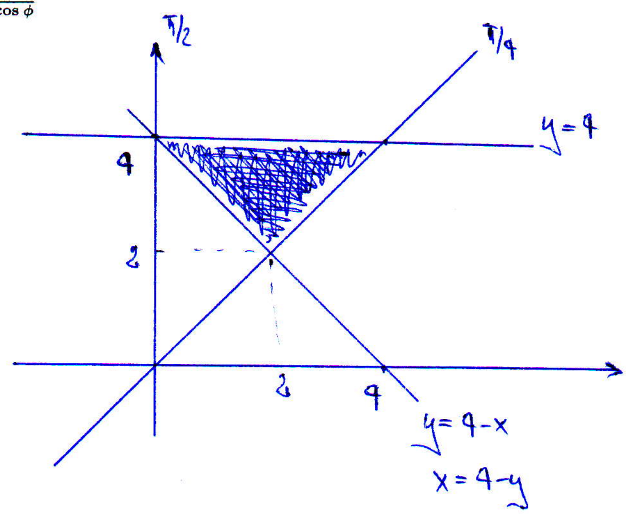
$$y + x = 4$$

$$y = 4 - x$$

$$r = \frac{4}{\sin\varphi}$$

$$r\sin\varphi = 4$$

$$y = 4$$



(ii) Gornji integral zapišite u Kartezijevim koordinatama. (1 bod)

1. način

$$\int_2^4 dy \int_{4-y}^{y} dx$$

2. način

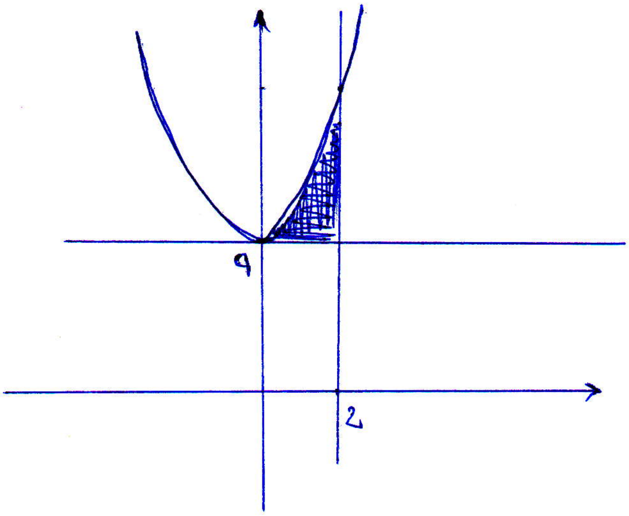
$$\int_0^2 dx \int_{4-x}^{4} dy + \int_2^4 dx \int_x^4 dy$$

(iii) Izračunajte taj integral. Možete koristiti oblik zadan pod (*i*) ili onaj koji ste dobili pod (*ii*). (1 bod)

$$\int_2^4 dy \int_{4-y}^{y} dx = \int_2^4 dy \; x\Big|_{4-y}^{y} = \int_2^4 dy\,(y-(4-y)) =$$

$$= \int_2^4 (2y-4)\,dy = \left(2\frac{y^2}{2} - 4y\right)\Big|_2^4 = 16-16-(4-8) = 4$$

3. Izračunajte volumen tijela omeđenog ravninama $x = 2$, $y = 4$, $z = 0$, $y = x^2 + 4$, $z = x^2$. (3 boda)



$z = f(x,y)$

$f(x,y) = x^2$

$$V = \int_0^2 dx \int_4^{x^2+4} x^2\, dy = \int_0^2 dx\ x^2 y \Big|_4^{x^2+4} = \int_0^2 dx\ x^2(x^2+4-4) =$$

$$= \int_0^2 x^4\, dx = \frac{x^5}{5}\Big|_0^2 = \frac{32}{5} - 0 = \frac{32}{5}$$

4. (i) Odredite opće rješenje diferencijalne jednadžbe

$$3y' = 6xe^{\frac{x}{3}} + y.$$

(2 boda)

$$3y' - y = 6xe^{x/3}$$

$$3y' - y = 0$$

$$3\frac{dy}{dx} = y \quad \Big| \ \frac{dx}{y}$$

$$3\int \frac{dy}{y} = \int dx$$

$$3\ln|y| = x + c$$

$$\ln|y| = \frac{x}{3} + c$$

$$y = e^{\frac{x}{3} + c}$$

$$y = ce^{x/3}$$

$$c = c(x), \quad y = c(x)e^{x/3}$$

$$3y' = 6xe^{x/3} + y$$

$$3\left(c'(x)e^{x/3} + c(x)\cdot\frac{1}{3}e^{x/3}\right) = 6xe^{x/3} + c(x)e^{x/3}$$

$$3c'(x)e^{x/3} + c(x)e^{x/3} = 6xe^{x/3} + c(x)e^{x/3}$$

$$3c'(x)e^{x/3} = 6xe^{x/3}$$

$$3c'(x) = 6x$$

$$c'(x) = 2x$$

$$c(x) = \int 2x\,dx = 2\frac{x^2}{2} + D = x^2 + D$$

$$y = (x^2 + D)e^{x/3}$$

(ii) Odredite ono partikularno rješenje jednadžbe iz (i) koje zadovoljava početni uvjet $y(3) = 0$. (1 bod)

$$y(3) = 0$$

$$0 = (9 + D)e$$

$$9 + D = 0$$

$$D = -9$$

$$y = (x^2 - 9)e^{x/3}$$

5. Odredite opće rješenje diferencijalne jednadžbe

$$y'' - 6y' + 9y = 3x + 7.$$

(3 boda)

$$y'' - 6y' + 9y = 0$$

$$\lambda^2 - 6\lambda + 9 = 0$$

$$\lambda_{1,2} = \frac{6 \pm \sqrt{36-36}}{2} = 3$$

$$y_0 = C_1 e^{3x} + C_2 x e^{3x}$$

$$Y = ax + b$$

$$Y' = a$$

$$Y'' = 0$$

$$Y'' - 6Y' + 9Y = 3x + 7$$

$$0 - 6a + 9(ax + b) = 3x + 7$$

$$-6a + 9ax + 9b = 3x + 7$$

$$\left.\begin{aligned} 9a &= 3 \\ -6a + 9b &= 7 \end{aligned}\right\}$$

$$a = \frac{1}{3}$$

$$-6 \cdot \frac{1}{3} + 9b = 7$$

$$9b = 9$$

$$b = 1 \quad \rightarrow \quad Y = \frac{1}{3}x + 1$$

$$y = y_0 + Y$$

$$y = C_1 e^{3x} + C_2 x e^{3x} + \frac{1}{3}x + 1$$