Rješenja Primjera 1. kol. iz Matematike 2.

1. (i) $\int f(x)\,dx = F(x)+C$ znači $(F(x)+C)' = f(x)$

tj. $\underline{F' = f}$.

(ii) Treba provjeriti je li $\left[\ln(x+\sqrt{x^2+1})\right]' = \frac{1}{\sqrt{x^2+1}}$.

Jeste (pogledajte izvod u lekciji 1.)

(iii) $y'(t) = -k\,y(t)$.

Tu je t vrijeme i $y(t)$ količina radioakt. materije u vrijeme t. Jednadžba proizlazi iz približne jednakosti $\Delta y(t) \approx -k \cdot y(t)\,\Delta t$ koja govori da je količina raspadnute materije u intervalu $[t, t+\Delta t]$ približno proporcionalna Δt i količini materije $y(t)$. Konstanta $k>0$ je karakteristika materije.

2. (i) $\int u\,dv = uv - \int v\,du$

(ii)
$$\int x e^{-2x}dx = \left[\begin{matrix} u = x, & du = dx \\ dv = e^{-2x}dx; & v = -\frac{1}{2}e^{-2x}\end{matrix}\right]$$
$$= x\cdot\left(-\frac{1}{2}e^{-2x}\right) - \int\left(-\frac{1}{2}e^{-2x}\right)dx$$
$$= \underline{-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C}$$

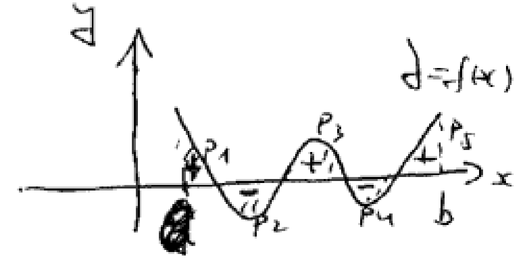
(iii) To je formula

$$\int u(x)\,v'(x)\,dx = u(x)\,v(x) - \int v(x)\,u'(x)\,dx$$

i izvede se iz $\left[u(x)\cdot v(x)\right]' = u'(x)\,v(x) + u(x)\,v'(x)$

(vidi lekciju 2.).

3\. (i)



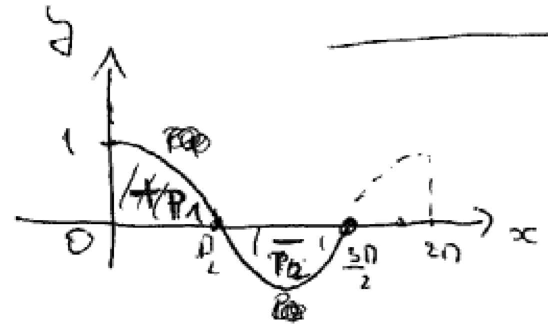
Integral pozitivne funkcije f na segmentu [a,b] jednak je površini između grafa funkcije i osi x.

$$P = \int_a^b f(x)\,dx$$

Integral funkcije f na segmentu [a,b] jednak je razlici površine ~~iznad~~ između grafa funkcije i osi x koji je iznad osi x i površine ispod osi x.

$$\int_a^b f(x)\,dx = P_1 - P_2 + P_3 - P_4 + P_5$$

(ii)



$$\int_0^{\frac{3\pi}{2}} \cos x\,dx = \sin x \Big|_0^{\frac{3\pi}{2}}$$

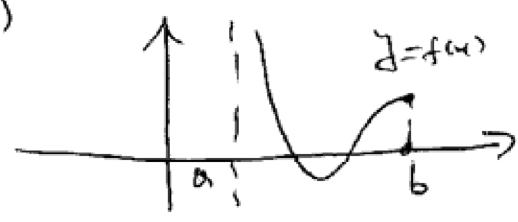
$$= \sin\frac{3\pi}{2} - \sin 0$$

$$= -1 - 0$$

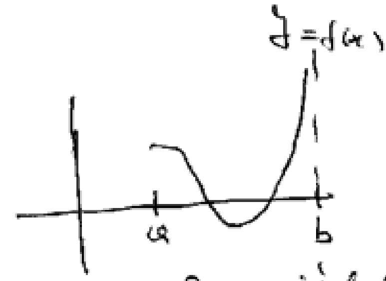
$$= \underline{\underline{-1}}$$

Dakle $P_1 - P_2 = -1$.

4\. (i)



f nije definirana u a



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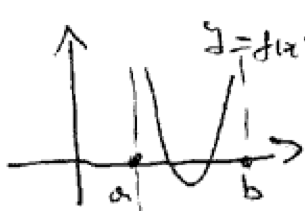
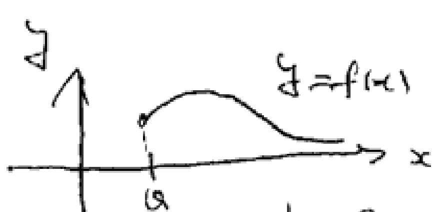
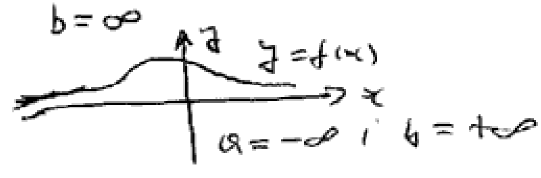


f nije definirana ni u a ni u b



$b = \infty$

$a = -\infty$



$a = -\infty$ i $b = +\infty$

(ii) Iz 2 (ii) dobijemo

$$\int_0^\infty x e^{-2x}\,dx = \left(-\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x}\right)\Big|_0^\infty = -\frac{1}{2}\,\frac{x}{e^{2x}}\Big|_0^\infty - \frac{1}{4}\cdot e^{-2x}\Big|_0^\infty$$

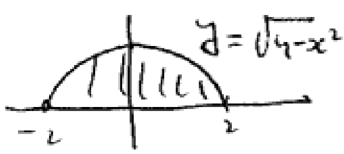
$$= (0-0) - \left(0 - \frac{1}{4}\right) = \frac{1}{4}$$

(pogledajte lekciju 4.)

(iii) $$\int_{-2}^{2} \sqrt{4-x^2}\,dx = \left[\begin{array}{ll} x = 2\sin t & ;\ x=2,\ t=\frac{\pi}{2} \\ dx = 2\cos t\,dt & ;\ x=-2;\ t=-\frac{\pi}{2} \end{array}\right]$$

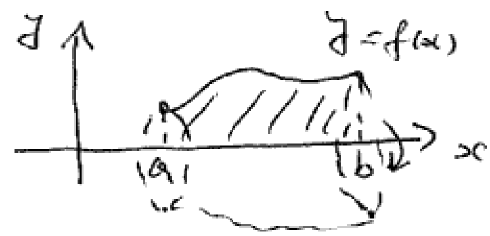
$$= \int_{-\pi/2}^{\pi/2} \sqrt{4-4\sin^2 t}\cdot 2\cos t\,dt = 4\int_{-\pi/2}^{\pi/2} \cos^2 t\,dt$$

(iv) $\int_{-2}^{2} \sqrt{4-x^2}\,dx = 2\pi$ jer je to polovica površine kruga polumjera $r=2$.



Može se i nastaviti računati integral (2 (iii)).

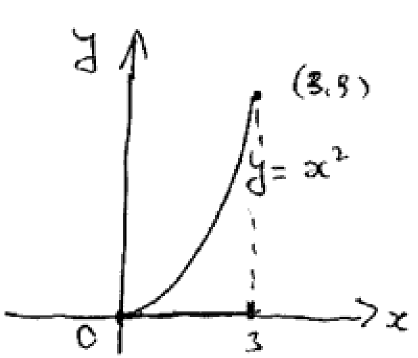
5. (i) $V = \pi \int_a^b f^2(x)\,dx$



(ii) Vidi lekciju 5.

(iii) $m = \int_a^b \rho(x)\,dx$

(iv)



$$m = \int_0^3 x^2\,dx = \frac{x^3}{3}\Big|_0^3 = \underline{\underline{9}}$$