OGLEDNI PRIMJER – 1. KOLOKVIJ – MAT. 2 – ZADACI

1. (i) Npr. $F_1(x) = \frac{-\cos 2x}{2} + 2x^2 + 3$

$F_2(x) = \frac{-\cos 2x}{2} + 2x^2 + 7,$

jer svaka od njih derivirajem daje $f(x)$, npr.

$$F_1'(x) = -\frac{1}{2}\cdot(\cos 2x)' + (2x^2)' + 3' =$$
$$= -\frac{1}{\cancel{2}}\cdot(-\sin 2x)\cancel{2} + 4x + 0 =$$
$$= \sin 2x + 4x = f(x)$$

(ii) Treba vidjeti da je $F'(x) = f(x)$

$$F'(x) = \frac{1}{4}\left(\ln\frac{2+x}{2-x}\right)' + (\sin 3)' =$$
$$= \frac{1}{4}\cdot\frac{1}{\frac{2+x}{2-x}}\cdot\left(\frac{2+x}{2-x}\right)' + 0 =$$
$$= \frac{1}{4}\cdot\frac{2-x}{2+x}\cdot\frac{1\cdot(2-x)-(2+x)(-1)}{(2-x)^2} =$$
$$= \frac{1}{4}\cdot\frac{2-\cancel{x}+2+\cancel{x}}{(2+x)(2-x)} = \frac{4}{4(4-x^2)} = \frac{1}{4-x^2} = f(x)$$

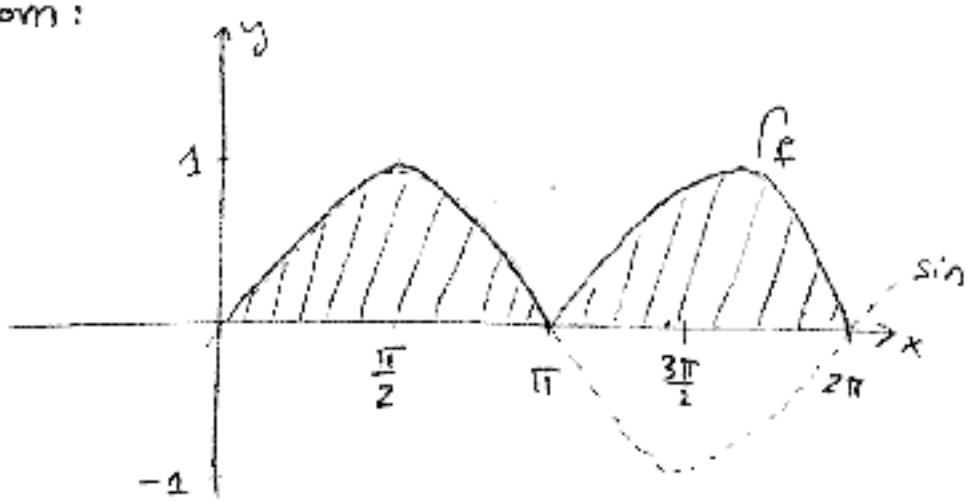
2. (i) $$\int\frac{\sin x}{\cos x+6}\,dx = \begin{bmatrix} \cos x+6=t \\ -\sin x\,dx = dt \end{bmatrix} = \int\frac{-dt}{t} = -\int\frac{dt}{t} =$$
$$= -\ln|t| + C = -\ln|\cos x+6| + C$$

(ii) $$\int e^x\sin x\,dx = \begin{bmatrix} u=e^x & dv=\sin x\,dx \\ du=e^x dx & v=-\cos x \end{bmatrix} =$$
$$= -e^x\cos x + \int e^x\cos x\,dx = \begin{bmatrix} u=e^x & dv=\cos x\,dx \\ du=e^x dx & v=\sin x \end{bmatrix} =$$
$$= -e^x\cos x + e^x\sin x - \int e^x\sin x\,dx$$

$$\Rightarrow 2\int e^x\sin x\,dx = e^x(\sin x - \cos x)$$

$$\Rightarrow \int e^x\sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + C$$

3. (i) $\int_0^{2\pi} |\sin x|\,dx$ brojčano odgovara površini lika omeđenog odozgo grafom funkcije $f(x)=|\sin x|$, odozdo x osi, a slijeva i zdesna pravcima $x=0$ i $x=2\pi$, redom:

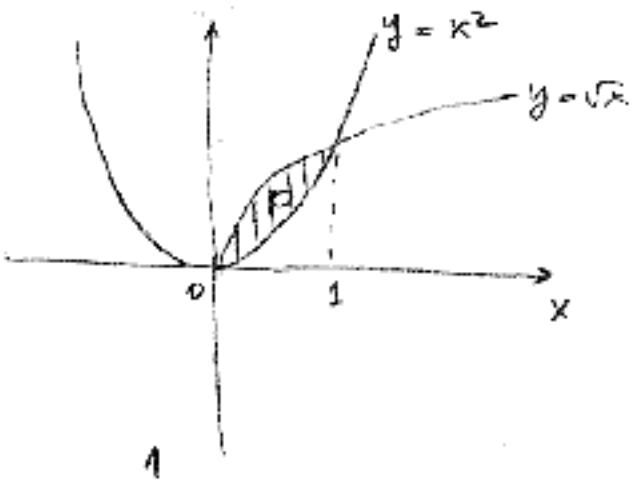


(ii) Sa slike vidimo da je

$$\int_0^{2\pi} |\sin x|\,dx = 2\int_0^{\pi} \sin x\,dx = 2\cdot(-\cos x)\Big|_0^{\pi} = 2(\underbrace{-\cos\pi}_{-1}+\underbrace{\cos 0}_{1}) =$$

$$= 2(1+1) = 2\cdot 2 = 4$$

4. (i)



Tražimo presjeke krivulja:

$x^2=\sqrt{x}$

$x^4=x$

$x^4-x=0$

$x(x^3-1)=0$

$\underline{x_1=0}$ $\quad x^3-1=0$

$x^3=1$

$\underline{x_2=1}$

$$\Rightarrow P=\int_0^1 (\underbrace{\sqrt{x}}_{x^{\frac{1}{2}}}-x^2)\,dx = \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^3}{3}\right)\Big|_0^1 =$$

$$=\left(\frac{2}{3}x\sqrt{x}-\frac{1}{3}x^3\right)\Big|_0^1 =$$

$$=\frac{2}{3}\cdot 1\cdot\sqrt{1}-\frac{1}{3}\cdot 1^3 = \frac{2}{3}-\frac{1}{3} = \boxed{\frac{1}{3}}$$

(ii) $$V = V_1 - V_2 = \pi \int_0^1 (\sqrt{x})^2 dx - \pi \int_0^1 (x^2)^2 dx =$$

$$= \pi \int_0^1 x\,dx - \pi \int_0^1 x^4 dx =$$

$$= \pi \int_0^1 (x - x^4)\,dx =$$

$$= \pi \cdot \left(\frac{x^2}{2} - \frac{x^5}{5}\right)\Big|_0^1 =$$

$$= \pi \left(\frac{1}{2} - \frac{1}{5}\right) =$$

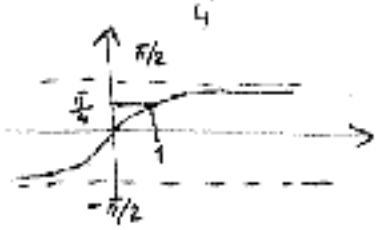
$$= \frac{5-2}{10}\pi =$$

$$= \boxed{\frac{3}{10}\pi}$$

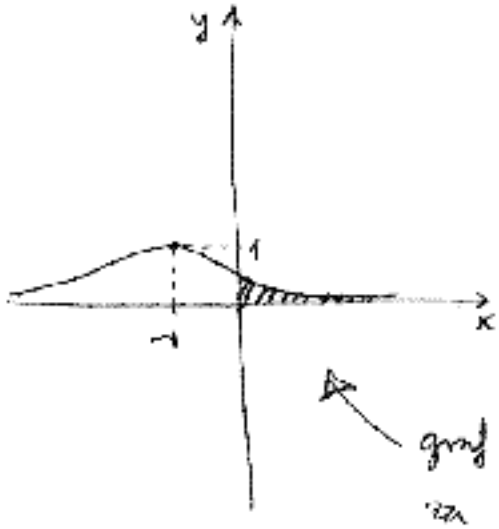
5. (i) $$\int_0^\infty \frac{1}{x^2+2x+2}dx = \lim_{b\to\infty} \int_0^b \frac{1}{\underbrace{x^2+2x+1}_{(x+1)^2}+1} dx = \lim_{b\to\infty} \int_0^b \frac{dx}{(x+1)^2+1^2} =$$

$$= \lim_{b\to\infty} [\operatorname{arctg}(x+1)]\Big|_0^b = \lim_{b\to\infty} (\cancel{\operatorname{arctg}(b+1)}^{\frac{\pi}{2}} - \underbrace{\operatorname{arctg} 1}_{=\frac{\pi}{4}}) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

koristimo formulu $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C$



(ii) Nacrtamo graf funkcije $f(x) = \frac{1}{x^2+2x+2}$:



Integral pod (i) računa površinu osjenčanog lika.

graf je dobiven translacijom grafa funkcije $\frac{1}{x^2}$ za 1 prema gore i za 1 ulijevo, jer je

$$f(x) = \frac{1}{(x+1)^2+1^2}$$