Rješenja

1. (i) $\cancel{\frac{\partial f}{\partial x} := \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}}$

$$\frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{\partial f}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

(ii) $\frac{\partial f}{\partial x} = 3x^2y^2$; $\frac{\partial f}{\partial y} = 2x^3y$

(iii) $\frac{\partial f}{\partial x}(1,2) = 3 \cdot 1^2 \cdot 2^2 = \underline{\underline{12}}$; $\frac{\partial f}{\partial y}(-1,2) = 2 \cdot (-1)^3 \cdot 2 = \underline{\underline{-4}}$

2. (i) $f(x_0+\Delta x, y_0+\Delta y) \approx f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)\,\Delta x + \frac{\partial f}{\partial y}(x_0,y_0)\,\Delta y$

Može i:

$$f(x,y) \approx f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)$$

(ii) $\Delta f(x_0,y_0) = f(x_0+\Delta x, y_0+\Delta y) - f(x_0,y_0)$

$$\Delta f(x_0,y_0) \approx \frac{\partial f}{\partial x}(x_0,y_0)\,\Delta x + \frac{\partial f}{\partial y}(x_0,y_0)\cdot\Delta y$$

(iii) $df(x_0,y_0) = \frac{\partial f}{\partial x}(x_0,y_0)\,dx + \frac{\partial f}{\partial y}(x_0,y_0)\,dy$

3. (i) ~~$\frac{\partial f}{\partial x}$~~ Ako je u (x_0,y_0) lokalni ekstrem onda je $\frac{\partial f}{\partial x}(x_0,y_0)=0$ i $\frac{\partial f}{\partial y}(x_0,y_0)=0$.

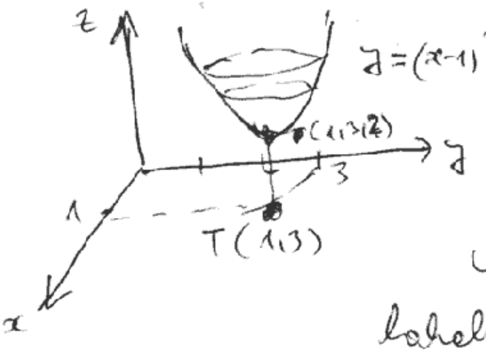
(ii) $\frac{\partial f}{\partial x} = 2x-2$; $\frac{\partial f}{\partial y}=2y-6$

Kritične točke su rješenja sustava $\frac{\partial f}{\partial x}=0$ i $\frac{\partial f}{\partial y}=0$,

tj. $2x-2=0$ i $2y-6=0$ tj. $x=1$ i $y=3$

Točka $T(1,3)$ je jedina kritična točka.

(iii) Vrijedi $f(x,y) = (x-1)^2+(y-3)^2+2$. Zato je graf:



Vidimo da je u $(1,3)$ ~~minimum~~ lokalni minimum i da je vrijednost minimuma 2.

4. (i) $\iint_D f(x,y)\,dx\,dy$ za pozitivnu funkciju f ima značenje obujma tijela između podskupa D i grafa funkcije f. Poželjna slika!!

(ii) $\iint_D f(x,y)\,dx\,dy = \int_a^b \left[\int_{\varphi(x)}^{\psi(x)} f(x,y)\,dy\right] dx$

ako je D zadano s: $a \le x \le b$, $\varphi(x) \le y \le \psi(x)$

Poželjna slika!

Tu je D: $0 \le x \le a$

$0 \le y \le b - \frac{b}{a}x$

jer je jednadžba dijagonale $\frac{x}{a}+\frac{y}{b}=1$ tj. $y = b - \frac{b}{a}x$

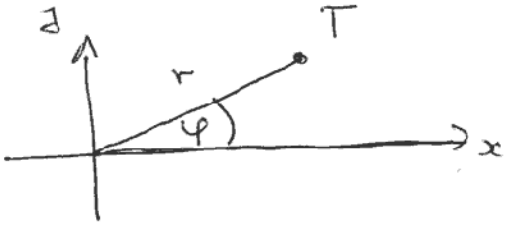
Zato je

$$\iint_D x\,dx\,dy = \int_0^a \Big[\int_{b-\frac{b}{a}x}^{b} x\,dy\Big]dx$$

$$= \int_0^a \Big(xy\Big|_{y=b-\frac{b}{a}x}^{y=b}\Big)dx = \int_0^a x\left(b-b+\frac{b}{a}x\right)dx$$

$$= \int_0^a \frac{b}{a}x^2\,dx = \frac{b}{a}\,\frac{x^3}{3}\Big|_0^a = \frac{b}{a}\cdot\frac{a^3}{3} = \frac{a^2b}{3}.$$

5. (i)

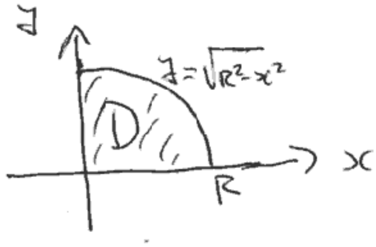


(ii)

$0 < \varphi < \frac{\pi}{2}$

$0 < r < +\infty$

(iii)



D je četvrtina kruga polumjera R u I kvadrantu. U polarnim koordinatama to je:

$0 \le \varphi \le \frac{\pi}{2}$ i $0 < r \le R$. Zato je

(uz $\sqrt{x^2+y^2} = r$)

$$\iint_D \sqrt{x^2+y^2} = \int_0^{\frac{\pi}{2}} \Big[\int_0^R r\cdot r\,dr\Big]d\varphi$$

$$= \int_0^{\pi/2} \Big(\frac{r^3}{3}\Big|_0^R\Big)d\varphi = \int_0^{\pi/2} \frac{R^3}{3}d\varphi = \frac{R^3}{3}\varphi\Big|_0^{\pi/2}$$

$$= \frac{R^3}{3}\cdot\frac{\pi}{2} = \frac{R^3\pi}{6}.$$