MATEMATIKA 1

PISMENI ISPITI 2002. - RJEŠENJA

20. veljače
11. svibnja
18. lipnja
2. srpnja
9. srpnja
11. rujna
25. rujna
1. listopada

MATEMATIKA I – 20. VELJAČE 2002.

RJEŠENJA

1. Odredite domenu funkcije $f(x) = \ln|\ln(15+2x-x^2)| + \dfrac{1}{(\sqrt{\pi} - 2\sqrt{\arcsin(x-1)})}$.

Rješenje. ln: $|\ln(15+2x-x^2)| > 0 \Rightarrow \ln(15+2x-x^2) \neq 0$

$\Rightarrow 15+2x-x^2 \neq 1$

$\Rightarrow x^2-2x-14 \neq 0$

$D = 4+56 = 60$

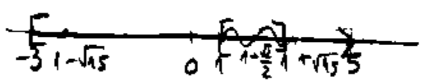
$\Rightarrow x_{1,2} = \dfrac{2 \pm 2\sqrt{15}}{2} = 1 \pm \sqrt{15}$

$\Rightarrow \boxed{x \notin \langle 1-\sqrt{15}, 1+\sqrt{15} \rangle}$

osim toga, mora biti $15+2x-x^2 > 0$

$x^2-2x-15 < 0$, $D = 4+60 = 64$

$x_{1,2} = \dfrac{2 \pm 8}{2} = 1 \pm 4 \Rightarrow x_1 = -3$, $x_2 = 5$



$\Rightarrow \boxed{x \in \langle -3, 5 \rangle}$

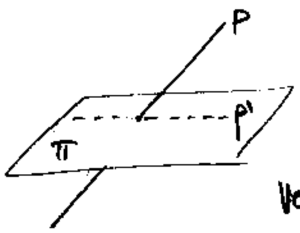
$\sqrt{\ }$: $\arcsin(x-1) \geq 0 \Rightarrow 1 \geq x-1 \geq 0 \Rightarrow 2 \geq x \geq 1 \Rightarrow \boxed{x \in [1,2]}$

razlomak: $\sqrt{\pi} \neq 2\sqrt{\arcsin(x-1)} \Rightarrow \dfrac{\pi}{4} \neq \arcsin(x-1)$ /sin $\Rightarrow \dfrac{\sqrt{2}}{2} \neq x-1 \Rightarrow \boxed{x \neq \dfrac{\sqrt{2}}{2}+1}$

$\Rightarrow \boxed{\mathcal{D}(f) = [1,2] \setminus \{1+\tfrac{\sqrt{2}}{2}\}}$

2. Odredite ortogonalnu projekciju pravca $p \ldots \begin{cases} x=1 \\ y=0 \end{cases}$ na ravninu $\Pi \ldots x+y+z=2$.

Rješenje.



Ideja: pronaći kroz p ravninu Π' koja je okomita na Π; projekcija p' pravca p na Π je presjek $\Pi \cap \Pi'$

Vektor smjera $\vec{s}$ pravca p dobije se kao vektorski produkt vektora normala ravnina $x=1$ i $y=0$: $\vec{s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \vec{k}$

Vektor normale $\vec{n}'$ ravnine Π' dobijemo kao vektorski produkt vektora normale ravnine Π i vektora smjera $\vec{s}$ pravca p: $\vec{n}' = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -\vec{i}+\vec{j}$

Vektor smjera $\vec{s}'$ projekcije p' dobijemo kao vektorski produkt vektora normale ravnine Π i Π': $\vec{s}' = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \vec{i}+\vec{j}-2\vec{k}$

Kao točku na p' možemo uzeti točku presjeka p i Π: $x=1, y=0 \Rightarrow 1+0+z=2 \Rightarrow z=1$

Sada je jednadžba pravca p': $\boxed{\dfrac{x-1}{1} = \dfrac{y}{1} = \dfrac{z-1}{-2}}$

3. $$\lim_{x\to -2} \frac{\sqrt{x+3}-1}{tg(\pi(x+3))} = \begin{bmatrix} \text{supst. } x+2=t \\ t\to 0\end{bmatrix} = \lim_{t\to 0}\frac{\sqrt{t+1}-1}{tg(\pi(t+1))} =$$

$$= \lim_{t\to 0}\frac{\sqrt{t+1}-1}{tg(\pi t+\pi)} = \lim_{t\to 0}\frac{\sqrt{t+1}-1}{\sin(\pi t+\pi)}\cdot\cos(\pi t+\pi) = \begin{bmatrix}\sin(x+\pi)=-\sin x\\ \cos(x+\pi)=-\cos x\end{bmatrix} =$$

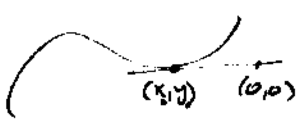
$$= \lim_{t\to 0}\frac{\sqrt{t+1}-1}{\sin \pi t}\cdot \cos\pi t = \lim_{t\to 0}\left(\frac{\sqrt{t+1}-1}{\pi t}\cdot\frac{\pi t}{\sin\pi t}\cdot\frac{\sqrt{t+1}+1}{\sqrt{t+1}+1}\cdot\cos\pi t\right) =$$

$$= \lim_{t\to 0}\left(\frac{\cancel{t}+\cancel{1}-\cancel{1}}{\pi\cancel{t}(\sqrt{t+1}+1)}\cdot \underbrace{\cos(\pi t)}_{1}\right)\cdot \underbrace{\cancel{\lim_{\substack{t\to 0\\(\pi t\to 0)}}\frac{\sin\pi t}{\sin\pi t}}}_{1} = \pi$$

$$= \frac{1}{2\pi}\cdot 1 = \boxed{\frac{1}{2\pi}}$$

4. Odredite jednadžbu tangente na $C \dots y=x^4-x+3$ koja prolazi ishodištem koord. sustava.

Rješenje.



Tražimo (x,y) na krivulji C tako da je pravac kroz (x,y) i $(0,0)$ tangenta na C u točki (x,y).

Prema formuli za jednadžbu pravca kroz 2 točke, t je pravac $y=\frac{y_0}{x_0}x$. Kako je (x_0,y_0) točka s krivulje, to mora biti

$y_0=x_0^4-x_0+3$.

S druge strane, koeficijent smjera tangente u točki (x_0,y_0) mora biti jednak $y'(x_0)$. Dakle mora biti $\frac{y_0}{x_0}=4x_0^3-1$, tj. $x_0^4-x_0+3=x_0(4x_0^3-1)$

$$x_0^4-\cancel{x_0}+3-4x_0^4+\cancel{x_0}=0$$

$$3=3x_0^4 \Rightarrow x_0^4=1 \Rightarrow x_0\in\{+1,-1\}$$

$x_0=-1 \Rightarrow y_0=1+1+3=5 \Rightarrow t_1 \dots \boxed{y=-5x}$

$x_0=1 \Rightarrow y_0=3 \Rightarrow t_2 \dots \boxed{y=3x}$

5. Ispitajte tok i nacrtajte graf funkcije $f(x)=\frac{x^2}{\ln x^2}$.

Rješenje. 1. $D(f)=?$ Treba biti: 1) $x^2>0 \Rightarrow x\neq 0$

2) $\ln x^2\neq 0 \Rightarrow x^2\neq 1 \Rightarrow x\neq 1$, $x\neq -1$

$\Rightarrow \boxed{D(f)=\mathbb{R}\setminus\{0,-1,1\}}$

2. $N(f)=?$ $x^2=0 \Rightarrow x=0$ ne može biti jer $0\notin D(f) \Rightarrow \boxed{N(f)=\emptyset}$

3. Asimptote: V.A. $x=1, x=-1$; u $x=0$ nemamo $\lim_{x\to 0} f(x)\to\pm\infty \to x=0$ nije V.A.

H.A. $\lim_{x\to\infty}\frac{x^2}{\ln x^2} \overset{L'H}{=} \lim_{x\to\infty}\frac{2\cancel{x}}{\frac{1}{x^2}\cdot 2\cancel{x}} = \lim_{x\to\infty} x^2 = \infty \Rightarrow$ nema

K.A. $k=\lim_{x\to\infty}\frac{x^{\cancel{2}}}{\cancel{x}\ln x^2} \overset{L'H}{=} \lim_{x\to\infty}\frac{1}{\frac{1}{x^2}\cdot 2x} = \lim_{x\to\infty}\frac{x^{\cancel{2}}}{2\cancel{x}} = \infty \Rightarrow$ nema

4. $f'(x) = \dfrac{2x\cdot\ln x^2 - \cancel{x^2}\cdot\frac{1}{\cancel{x^2}}\cdot 2x}{(\ln x^2)^2} = \dfrac{2x(\ln x^2-1)}{(\ln x^2)^2} = 0 \Rightarrow$ $x=0$ ne može biti, jer $x \notin D(f)$

$\ln x^2 = 1 \Rightarrow x^2 = e$
$\Rightarrow \underline{x_1 = -\sqrt{e},\ x_2 = \sqrt{e}}$
kritične točke

5. $f''(x) = \dfrac{\left(2(\ln x^2-1) + 2\cancel{x}\left(\frac{1}{\cancel{x^2}}\cdot 2\cancel{x}\right)\right)\cdot(\ln x^2)^2 - 2\cancel{x}(\ln x^2-1)\cdot 2(\ln x^2)\cdot\frac{1}{\cancel{x^2}}2x}{(\ln x^2)^4}$

$f''(x) = \dfrac{(2\ln x^2 \cancel{-2} \cancel{+2})(\ln x^2)^{\cancel{2}} - 8(\cancel{\ln x^2})(\ln x^2-1)}{(\ln x^2)^{\cancel{4}}}$

$f''(x) = \dfrac{2(\ln x^2)^2 - 8(\ln x^2-1)}{(\ln x^2)^3}$

$f''(\pm\sqrt{e}) = \dfrac{2(\ln e)^2 - 8(\ln e - 1)}{(\ln e)^3} = \dfrac{2-8\cdot(1-1)}{1^3} = 2 > 0 \Rightarrow$ u $\sqrt{e}$ i $-\sqrt{e}$ se postiže lok. minimum

$f(\pm\sqrt{e}) = \dfrac{e}{\ln e} = e \Rightarrow \boxed{(-\sqrt{e}, e),\ (\sqrt{e}, e)}$

Točke infleksije: uz supstituciju $\ln x^2 = t$ rješavamo $2t^2 - 8t + 8 = 0$
$t^2 - 4t + 4 = 0$
$(t-2)^2 = 0 \Rightarrow t = 2$
$\ln x^2 = 2 \Rightarrow x^2 = e^2 \Rightarrow \underline{x_1 = -e}$, $\underline{x_2 = e}$

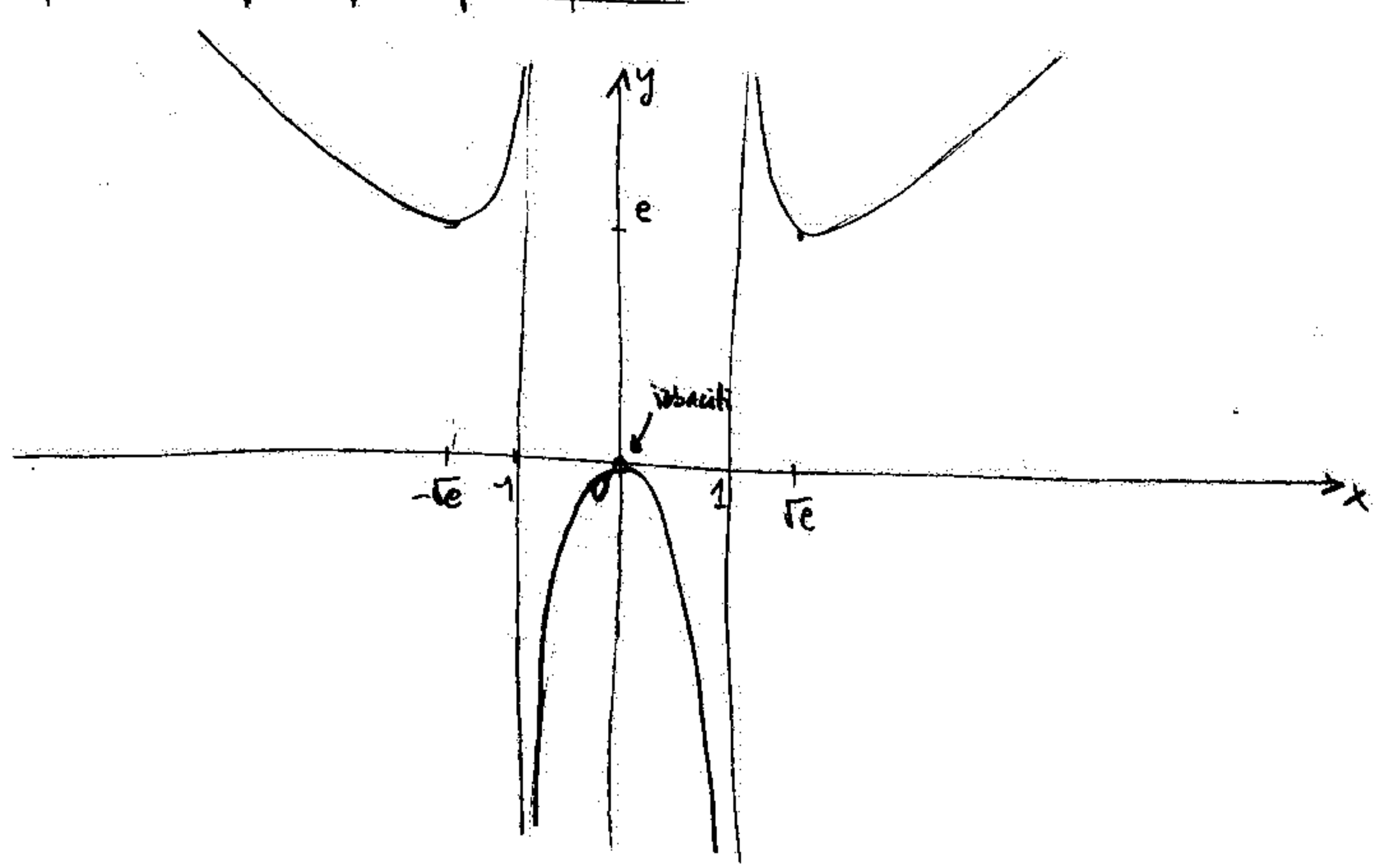
6. $f'(x) < 0 \Rightarrow x(\ln x^2 - 1) < 0$

Dobije se $\underline{x \in \langle -\infty, -\sqrt{e}\rangle \cup \langle 0, \sqrt{e}\rangle \setminus \{1\}}$ pad

$f'(x) > 0 \Rightarrow$ dobije se $\underline{x \in \langle -\sqrt{e}, 0\rangle \setminus \{-1\} \cup \langle \sqrt{e}, \infty\rangle}$ rast

$D(f)$	$\langle -\infty, -\sqrt{e}\rangle$	$-\sqrt{e}$	$\langle -\sqrt{e}, -1\rangle$	-1	$\langle -1, 0\rangle$	0	$\langle 0, 1\rangle$	1	$\langle 1, \sqrt{e}\rangle$	$\sqrt{e}$	$\langle \sqrt{e}, \infty\rangle$
f'	$-$	0	$+$		$+$		$-$		$-$	0	$+$
f	↘	MIN	↗		↗		↘		↘	MIN	↗

7.



MATEMATIKA 1.

11.5.2002.

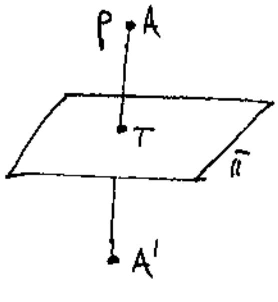
RJEŠENJA ZADATAKA

1. Odredite točku simetričnu točki $A(1,2,0)$ obzirom na ravninu $2x+3y-4z+21=0$.

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \| \ \| \ \|$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x_A \ y_A \ z_A$

Rješenje.



$A'=?$ ideja je da kroz točku A povučemo pravac okomit na ravninu π – presjek tog pravca s ravninom je točka T, a onda je (uz $A'=(x_{A'},y_{A'},z_{A'})$, $T=(x_T,y_T,z_T)$)

$$\frac{x_A+x_{A'}}{2}=x_T, \quad \frac{y_A+y_{A'}}{2}=y_T, \quad \frac{z_A+z_{A'}}{2}=z_T \ \text{(polovište)}$$

Ako označimo koeficijent smjera pravca p sa $\vec{s}$, očito je $\vec{s}=\vec{n}$, gdje je $\vec{n}$ vektor normale od π

$$\Rightarrow \vec{s}=2\vec{i}+3\vec{j}-4\vec{k}, \ p\sim? \quad p\ldots \ \frac{x-1}{2}=\frac{y-2}{3}=\frac{z}{-4} \rightarrow \begin{array}{l} x=2t+1 \\ y=3t+2 \\ z=-4t \end{array}$$

Uvrstimo tu točku u jednadžbu ravnine (tražimo $T=p\cap\pi$):

$$2(2t+1)+3(3t+2)-4(-4t)+21=0 \Rightarrow 29t=-29 \Rightarrow t=-1 \Rightarrow x_T=-1,\ y_T=-1,\ z_T=4$$

$$\Rightarrow x_{A'}=2\cdot(-1)-1 \rightarrow \boxed{x_{A'}=-3} \ / \ y_{A'}=2\cdot(-1)-2 \Rightarrow \boxed{y_{A'}=-4} \ / \ z_{A'}=2\cdot 4 \Rightarrow \boxed{z_{A'}=8}$$

$$\Rightarrow \boxed{A'=(-3,-4,8)}$$

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2. Izračunajte $\lim\limits_{x\to-\infty} f(x)+\lim\limits_{x\to\infty} f(x)$ bez upotrebe L'Hospitala ako je $f(x)=\sqrt{x^2+2x+3}-\sqrt{x^2-2x+3}$.

Rješenje.

$$f(x)=\left(\sqrt{x^2+2x+3}-\sqrt{x^2-2x+3}\right)\cdot\frac{\sqrt{x^2+2x+3}+\sqrt{x^2-2x+3}}{\sqrt{x^2+2x+3}+\sqrt{x^2-2x+3}}=$$

antiracionalizacija

$$=\frac{\left(\sqrt{x^2+2x+3}\right)^2-\left(\sqrt{x^2-2x+3}\right)^2}{\sqrt{x^2+2x+3}+\sqrt{x^2-2x+3}}=\frac{\cancel{x^2}+2x+\cancel{3}-\cancel{x^2}+2x-\cancel{3}}{\sqrt{x^2+2x+3}+\sqrt{x^2-2x+3}}=$$

$$=\frac{4x}{\sqrt{x^2}\sqrt{1+\frac{2}{x}+\frac{3}{x^2}}+\sqrt{x^2}\cdot\sqrt{1-\frac{2}{x}+\frac{3}{x^2}}}=\frac{4\cancel{x} \ \pm \text{ovisno o } x\to-\infty \text{ ili } x\to+\infty}{|\cancel{x}|\left(\sqrt{1+\underbrace{\frac{2}{x}+\frac{3}{x^2}}_{0}}+\sqrt{1-\underbrace{\frac{2}{x}+\frac{3}{x^2}}_{0}}\right)}$$

u limesu

$$\Rightarrow \lim_{x\to-\infty} f(x)+\lim_{x\to\infty} f(x)=-\frac{4}{1+1}+\frac{4}{1+1}=-2+2=\boxed{0}$$

3. Odredite domenu funkcije $f(x)=\ln\left(\dfrac{1}{(\sqrt{x-2}-1)(\sqrt{x-3}-2)}\right)$

Rješenje. Mora biti $\left.\begin{array}{l} x-2\geq 0 \\ x-3\geq 0 \end{array}\right\} \Rightarrow \boxed{x\geq 3}_{(*)}$ (zbog korijena)

Zbog f-je ln je $\dfrac{1}{(\sqrt{x-2}-1)(\sqrt{x-3}-2)}>0$, tj.

$$(\sqrt{x-2}-1)(\sqrt{x-3}-2)>0$$

Imamo 2 mogućnosti: a) $\begin{array}{l}\sqrt{x-2}-1>0\\ \sqrt{x-3}-2>0\end{array}$ ili b) $\begin{array}{l}\sqrt{x-2}-1<0\\ \sqrt{x-3}-2<0\end{array}$

a) $\Rightarrow \sqrt{x-2}>1 \,/()^2$, $\sqrt{x-3}>2\,/()^2$ $\Rightarrow x-2>1\Rightarrow \boxed{x>3}$, $x-3>4\Rightarrow\boxed{x>7}$ $\Rightarrow \boxed{x>7}$

b) $\sqrt{x-2}<1\,/()^2$, $\sqrt{x-3}<2\,/()^2$ $\Rightarrow x-2<1\Rightarrow\boxed{x<3}$, $x-3<4\Rightarrow\boxed{x<7}$ $\Rightarrow\boxed{x<3}$

(**) $\boxed{x\in\langle-\infty,3\rangle\cup\langle 7,\infty\rangle} \Leftarrow$

Kada pogledamo (*) i (**), vidimo da je $\boxed{\mathcal{D}(f)=\langle 7,\infty\rangle}$

4. Na krivulji $y=x^3-3x^2+3x-4$ pronađite tangentu okomitu na pravac $x=2002$.

Rješenje. Pravci okomiti na pravac $x=2002$ su oni oblika $y=c$, $c\in\mathbb{R}$ konstanta

$\Rightarrow y=0\cdot x+c \Rightarrow \underline{\text{koef. smjera}=0}$

$y'(x)=3x^2-6x+3=0 \Rightarrow x^2-2x+1=0 \Rightarrow \boxed{x=1} \Rightarrow y=1-3+3-4 \Rightarrow \boxed{y=-3}$

$\Rightarrow \boxed{y=-3}$ je tražena tangenta

5. Ispitajte tok i nacrtajte graf f-je $f(x)=x\cdot(\ln x)^2$

Rješenje. 1) $\mathcal{D}(f)=\langle 0,\infty\rangle$ (zbog ln f-je)

2) $N(f)=?$ $\cancel{x=0}$ ili $\ln x=0$ $(0\notin D(f)!)$ $\Rightarrow x=1 \Rightarrow N(f)=\{1\}$

3) V.A. nema, jer $\lim\limits_{x\to 0_+} x(\ln x)^2=\lim\limits_{x\to 0_+}\dfrac{(\ln x)^2}{\frac{1}{x}} \overset{L'H}{=} \lim\limits_{x\to 0_+}\dfrac{2\ln x\cdot\frac{1}{\cancel{x}}}{-\frac{\cancel{x}}{x^2}}=-2\lim\limits_{x\to 0_+}\dfrac{\ln x}{\frac{1}{x}} \overset{L'H}{=}$

$$=-2\lim_{x\to 0_+}\frac{\frac{1}{\cancel{x}}}{-\frac{\cancel{1}}{x^2}}=2\lim_{x\to 0_+}x=0$$

K.A. nema

H.A. $\lim\limits_{x\to+\infty}f(x)=+\infty$, nema

4) $f'(x)=(\ln x)^2+\cancel{x}\cdot 2\ln x\cdot\frac{1}{\cancel{x}}=\ln x(\ln x+2)=0 \Rightarrow \ln x=0$ ili $\ln x+2=0$

$\underline{x_1=1}$ $\underline{x_2=e^{-2}}$ kandidati za ekstrem

rast-pad: $\underline{\text{rast}}$ $f'(x)>0 \Rightarrow \begin{array}{l}\ln x>0\\ \ln x+2>0\end{array}$ ili $\begin{array}{l}\ln x<0\\ \ln x+2<0\end{array}$

$\ln x>0\Rightarrow\boxed{x>1}$ ili $\ln x<-2\Rightarrow\boxed{x<\frac{1}{e^2}}$

$\Rightarrow$ rast na $\langle 0, \frac{1}{e^2} \rangle \cup \langle 1, \infty \rangle$

$\Rightarrow$ pad na $\langle \frac{1}{e^2}, 1 \rangle$

5) $f''(x) = 2\ln x \cdot \frac{1}{x} + 2 \cdot \frac{1}{x} = \frac{2}{x}(\ln x + 1) = 0 \Rightarrow \ln x = -1 \Rightarrow \boxed{x = \frac{1}{e} \text{ T. infleksije}}$

$f''(1) = 2(\ln 1 + 1) = 2 > 0 \Rightarrow (1, f(1)) = \boxed{(1,0) \text{ MIN}}$

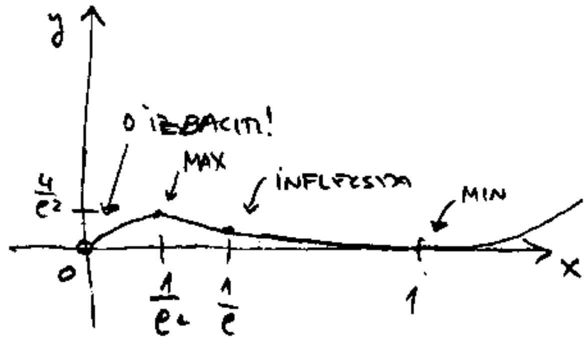
$f''(\frac{1}{e^2}) = 2e^2(-2+1) = -2e^2 < 0 \Rightarrow (\frac{1}{e^2}, f(\frac{1}{e^2})) = \boxed{(\frac{1}{e^2}, \frac{4}{e^2}) \text{ MAX}}$

6) tok:

$D(f)$	$\langle 0, \frac{1}{e^2} \rangle$	$\frac{1}{e^2}$	$\langle \frac{1}{e^2}, 1 \rangle$	1	$\langle 1, \infty \rangle$
f'	$+$	0	$-$	0	$+$
f	$\nearrow$	MAX	$\searrow$	MIN	$\nearrow$

$\Rightarrow$

7) graf



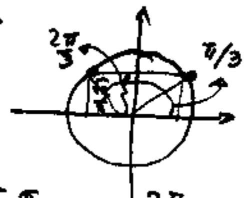
MATEMATIKA I / 18.6.2002. / RJEŠENJA

1. Odredite domenu funkcije $f(x)=\sqrt{\sin(\arccos x)-\frac{\sqrt{3}}{2}}$.

Rješenje.

$\sin(\arccos x)-\frac{\sqrt{3}}{2}\geq 0$ i $\boxed{x\in[-1,1]}$ (zbog arccos)

$\sin(\arccos x)\geq\frac{\sqrt{3}}{2}$



$\Rightarrow$

$$\arccos x\in\bigcup_{k\in\mathbb{Z}}\left[\frac{\pi}{3}+2k\pi,\frac{2\pi}{3}+2k\pi\right]$$

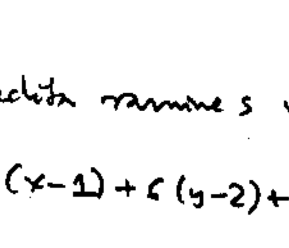
Kako je skup vrijednosti funkcije arccos jednak $[0,\pi]$, to mora biti $k=0$, tj. $\arccos x\in\left[\frac{\pi}{3},\frac{2\pi}{3}\right]$

$\frac{\pi}{3}\leq\arccos x\leq\frac{2\pi}{3}\Rightarrow\boxed{x\in\left[-\frac{1}{2},\frac{1}{2}\right]}$

$\Rightarrow\boxed{\mathcal{D}(f)=\left[-\frac{1}{2},\frac{1}{2}\right]}$

2. Odredite jednadžbu ravnine koja sadrži točke $T_1(1,2,0)$ i $T_2(2,3,1)$, a okomita je na ravninu $2x+3y-4z=0$.

Rješenje.



$\Pi_1 \ldots 2x+3y-4z=0$

$\Pi_2=?$

$\vec{n}_2\perp\vec{n}_1,\ \overrightarrow{T_1T_2}=\vec{i}+\vec{j}+\vec{k}$

$$\Rightarrow\vec{n}_2=\vec{n}_1\times\overrightarrow{T_1T_2}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\1&1&1\\2&3&-4\end{vmatrix}=-7\vec{i}+6\vec{j}+\vec{k}$$

Jednadžba ravnine s vektorom normale $\vec{n}_2$, a s točkom $T_1(1,2,0)$ glasi

$-7(x-1)+6(y-2)+1\cdot(z-0)=0\ /\cdot(-1)\Rightarrow\boxed{7x-6y-z+5=0}$

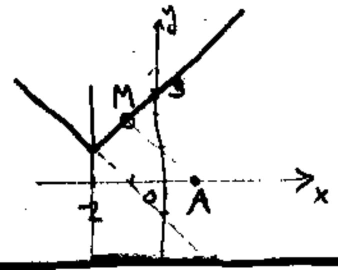
3. Koristeći diferencijalni račun na krivulji $y=|x+2|+1$ nađite točku najbližu točki $A(1,0)$.

Rješenje.

$x+2=0$

$\underline{x=-2}$

lomna točka



$x\leq-2$

$y=-x-2+1$

$\boxed{y=-x-1}$

$x\geq-2$

$y=x+2+1$

$\boxed{y=x+3}$

$M = ?$

Gledamo dvije mogućnosti:

1) $x \le -2$

$y = -x-1$

Udaljenost A od točke $T(x,-x-1)$ na $y=-x-1$ je $f(x) = \sqrt{(x-1)^2+(-x-1-0)^2} =$

$= \sqrt{2x^2+2}$. $f'(x) = \dfrac{4x}{2\sqrt{2x^2+2}} = 0 \Rightarrow x=0$ što nije u domeni

Kako je funkcija f padajuća na $\langle -\infty, 0]$ očito je minimum u $x=-2$ i iznosi

$f(-2) = \sqrt{2\cdot 4+2} = \sqrt{10}$

2) $x \ge -2$

$y = x+3$

$f(x) = \sqrt{(x-1)^2+(x+3)^2} = \sqrt{2x^2+4x+10} \Rightarrow f'(x) = \dfrac{4x+4}{2\sqrt{2x^2+4x+10}} = 0 \Rightarrow \boxed{x=-1}$

$f(-1) = \sqrt{2-4+10} = \sqrt{8} = 2\sqrt{2}$

$\Rightarrow$ Minimalna udaljenost točke A do $y=|x+1|+1$ je $\underline{2\sqrt{2}}$, i to je udaljenost od $\underline{(-1,2)}$

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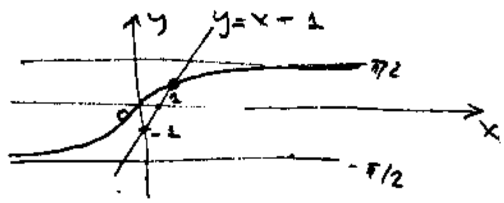
4. Ispitajte tok i nacrtajte graf funkcije $f(x) = \text{arctg}\, x - x + 1$.

<u>Rješenje.</u> 1) $\mathcal{D}(f) = \mathbb{R}$

2) $\mathcal{N}(f) = ?$ $\quad \text{arctg}\, x = x-1$



Postoji jedna nultočka x_0, $x_0 > 1$.

3) nema vertikalnih asimptota;

horizontalne asimptote: $y = \lim\limits_{x\to-\infty} (\underbrace{\text{arctg}\, x}_{-\frac{\pi}{2}} \underbrace{-x}_{\infty} + 1)$ nema

$y = \lim\limits_{x\to\infty} (\underbrace{\text{arctg}\, x}_{\frac{\pi}{2}} \underbrace{-x}_{\infty} + 1)$ nema

kose asimptote: $k = \lim\limits_{x\to\infty} \dfrac{\text{arctg}\, x - x + 1}{x} = \lim\limits_{x\to\infty} \left(\dfrac{\text{arctg}\, x}{x} - 1 + \dfrac{1}{x}\right) =$

$= \left(\lim\limits_{x\to\infty} \dfrac{\text{arctg}\, x}{x}\right) - 1 + \underbrace{\boxed{\lim\limits_{x\to\infty} \frac{1}{x}}}_{=0} \overset{L'H}{=} \lim\limits_{x\to\infty} \dfrac{\frac{1}{1+x^2}}{\cancel{1}} - 1 + 0 = -1$

$l = \lim\limits_{x\to\infty} (\text{arctg}\, x - \cancel{x} + 1 + \cancel{1x}) = \frac{\pi}{2} + 1$

$y = -x + \frac{\pi}{2} + 1$ je desna kosa asimptota

Za lijevu kosu asimptotu k je isti, a $l = \lim\limits_{x\to-\infty} (\text{arctg}\, x + 1) = -\frac{\pi}{2} + 1 \Rightarrow \underline{y = -x - \frac{\pi}{2} + 1}$

4) $f'(x) = \dfrac{1}{1+x^2} - 1 = 0 \Rightarrow 1+x^2 = 1 \Rightarrow \underline{x=0}$ kritična točka

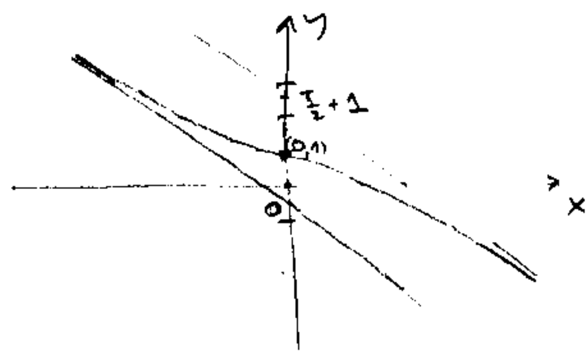
5) $f''(x) = \dfrac{-1}{(1+x^2)^2} \cdot 2x = 0 \qquad f''(0) = 0 \Rightarrow (f(0) = \text{arctg}\, 0 - 0 + 1 = 1)$ $\underline{(0,1)}$ točka infleksije

6) tok:

rast $f'(x) > 0 \Rightarrow \frac{1}{1+x^2} - 1 > 0 \Rightarrow 1 > 1 + x^2 \Rightarrow x^2 < 0 \Rightarrow x \in \emptyset$

$\Rightarrow$ funkcija pada na cijelom području definicije

7) graf



5. Bez upotrebe L'Hospitalovog pravila izračunajte $\lim\limits_{x \to 0} \frac{1-4\cos x + 3\cos^2 x}{x^2}$.

Rješenje. $\lim\limits_{x \to 0} \frac{1-4\cos x+3\cos^2 x}{x^2} = \lim\limits_{x \to 0} \frac{1-4\cos x + 3(1-\sin^2 x)}{x^2} =$

$= \lim\limits_{x \to 0} \left(\frac{4-4\cos x}{x^2} - 3 \cdot \left(\frac{\sin x}{x}\right)^2\right) = \lim\limits_{x \to 0} \left(\frac{4(1-\cos x)}{x^2}\right) - 3\Big(\underbrace{\lim\limits_{x \to 0} \frac{\sin x}{x}}_{=1}\Big)^2 =$

$= 4 \lim\limits_{x \to 0} \frac{2\sin^2 \frac{x}{2}}{x^2} - 3 = 8 \lim\limits_{x \to 0} \left(\frac{\sin^2 \frac{x}{2}}{\frac{x^2}{4}} \cdot \frac{1}{4}\right) - 3 = 2\left(\lim\limits_{\frac{x}{2} \to 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2 - 3 =$

koristimo formulu $1-\cos x = 2\sin^2 \frac{x}{2}$

$= 2 - 3 = \boxed{-1}$

MATEMATIKA I – 2.7.2002.

RJEŠENJA

1. Odredite domenu funkcije $f(x)=\sqrt{\frac{x^2}{9-x^2}\left(\pi^2-(6\arcsin x)^2\right)}$.

Rješenje. $\frac{x^2}{9-x^2}\cdot\left(\pi^2-(6\arcsin x)^2\right)\geq 0$

$x^2\geq 0 \quad \forall x\in\mathbb{R}$

$\Rightarrow$ ostaje uvjet $\frac{\pi^2-(6\arcsin x)^2}{9-x^2}\geq 0$

1) $\begin{cases}\pi^2-(6\arcsin x)^2\geq 0\\ 9-x^2>0\end{cases}$ $\longrightarrow (\pi-6\arcsin x)(\pi+6\arcsin x)\geq 0$

$9>x^2$

$|x|<3$

a) $\pi-6\arcsin x\geq 0$, $\pi+6\arcsin x\geq 0$

$-\frac{\pi}{6}\leq\arcsin x\leq\frac{\pi}{6}$

$\Rightarrow x\in\left[-\frac{1}{2},\frac{1}{2}\right]$

b) $\pi-6\arcsin x\leq 0$, $\pi+6\arcsin x\leq 0$

$\arcsin x\leq-\frac{\pi}{6}$, $\arcsin x\geq\frac{\pi}{6}$

$\Rightarrow x\in\emptyset$

$\boxed{x\in\left[-\frac{1}{2},\frac{1}{2}\right]}$

2) $\begin{cases}\pi^2-(6\arcsin x)^2\leq 0\\ 9-x^2<0\end{cases}$ $\longrightarrow$ ovo ni ne treba rješavati, jer je domena arcsin funkcije $[-1,1]$, a zahtjev (*) je $x\in\langle-\infty,-3\rangle\cup\langle 3,\infty\rangle$

$9<x^2$

$|x|>3$ (*)

$\Rightarrow \boxed{\mathcal{D}(f)=\left[-\frac{1}{2},\frac{1}{2}\right]}$

---

2. Izračunajte $\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}\cdot\frac{1}{\cos x}-x\,\mathrm{tg}\,x\right)$ bez upotrebe L'Hospitalovog pravila.

Rješenje. uvodimo supstituciju $x-\frac{\pi}{2}=t$

$\cos x=\cos\left(\frac{\pi}{2}+t\right)=-\sin t$

$\mathrm{tg}\,x=\mathrm{tg}\left(\frac{\pi}{2}+t\right)=\frac{\sin\left(\frac{\pi}{2}+t\right)}{\cos\left(\frac{\pi}{2}+t\right)}=\frac{\cos t}{-\sin t}$

Limes postaje $\lim\limits_{t\to 0}\left(-\frac{\pi}{2}\cdot\frac{1}{\sin t}-\left(t+\frac{\pi}{2}\right)\cdot\frac{\cos t}{-\sin t}\right)=$

$=\lim\limits_{t\to 0}\left(-\frac{\pi}{2}\cdot\boxed{\frac{1}{\sin t}}+\frac{t\cos t}{\sin t}+\frac{\pi}{2}\cdot\boxed{\frac{\cos t}{\sin t}}\right)=$

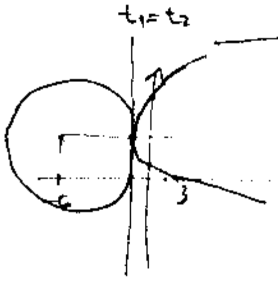
$=\lim\limits_{t\to 0}\left(\frac{\pi}{\cancel{2}}\cdot\frac{1}{\sin t}\underbrace{(\cos t-1)}_{=-\cancel{2}\sin^2\frac{t}{2}}+\frac{t\cos t}{\sin t}\right)=$

$=-\pi\lim\limits_{t\to 0}\frac{\sin^{\cancel{2}}\frac{t}{2}}{2\cancel{\sin\frac{t}{2}}\cdot\cos\frac{t}{2}}+\lim\limits_{t\to 0}\left(\frac{t}{\sin t}\cdot\cos t\right)=-\pi\lim\limits_{t\to 0}\frac{\sin\frac{t}{2}}{\cos\frac{t}{2}}+1=\boxed{1}$

3. Nađite kut među krivuljama: $x^2+12x+y^2-4y+15=0$,
$$x-y^2+4y-3=0$$

Rješenje. Tražimo presječne točke dviju krivulja:

$$(x+6)^2-36+(y-2)^2-4+15=0 \rightsquigarrow (x+6)^2+(y-2)^2=25$$
$$x-(y-2)^2+4-3=0 \rightsquigarrow x-(y-2)^2=-1 \quad /+$$



$$x^2+12x+36+x=24$$
$$x^2+13x+12=0$$
$$x_{1,2}=\frac{-13\pm\sqrt{169-48}}{2}=\frac{-13\pm 11}{2}$$
$$x_1=-12 \Rightarrow (y-2)^2=-12+1 \;\lightning$$
$$x_2=-1 \Rightarrow \cancel{-1}-(y-2)^2=\cancel{-1}$$
$$\underline{y=2}$$
$$\boxed{(-1,2)}$$

• $t_1 \ldots$

$(x+6)^2+(y-2)^2=25$

Implicitno deriviramo i dobijemo $y'=-\dfrac{x+6}{y-2}$

$$y'(-1,2)=-\frac{-1+6}{2-2}=\infty=\operatorname{tg}\alpha_1$$
$$\underline{\alpha_1=\frac{\pi}{2}}$$

• $t_2 \ldots$

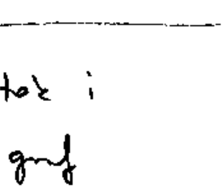
$x-(y-2)^2+1=0$ Implicitno deriviramo i dobijemo

$$y'=\frac{1}{2(y-2)} \Rightarrow y'(-1,2)=\frac{-1}{2(2-2)}=\infty=\operatorname{tg}\alpha_2 \Rightarrow \underline{\alpha_2=\frac{\pi}{2}}$$

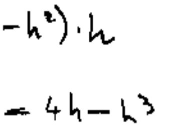
$\Rightarrow$ tangente (obje) su pravci $x_1=x_2=-1$, pa je kut među krivuljama je 0

4. Iz kruga polumjera $r=2$ izrežite kružni isječak koji savijanjem daje stožac maksimalnog obujma.

Rješenje.



$$V_S=\frac{\pi x^2}{3}\cdot h$$
$$I=\frac{r\cdot l}{2}$$



$$2\pi x=l \qquad h^2+x^2=4$$
$$x=\frac{l}{2\pi} \qquad x^2=4-h^2$$

$$\Rightarrow V_S=\frac{\pi}{3}\cdot(4-h^2)\cdot h$$

$$f(h)=(4-h^2)h=4h-h^3$$
$$f'(h)=4-3h^2=0 \Rightarrow h^2=\frac{4}{3} \Rightarrow \underline{h=\frac{2\sqrt3}{3}}$$
$$x^2=4-\frac{4}{3}=\frac{8}{3} \Rightarrow$$
$$\Rightarrow x=\frac{2\sqrt2\cdot\sqrt3}{3}=\frac{2\sqrt6}{3}$$

$$\text{iz } I=\frac{r^2\pi\alpha}{360^\circ}=\frac{rl}{2} \quad \Leftarrow \boxed{l=\frac{4\pi\sqrt6}{3}}$$
$$\cancel{2}\cdot\frac{\cancel{\pi}}{360^\circ}\cdot\alpha=\frac{1}{\cancel{2}}\cdot\frac{\cancel{4}\cancel{\pi}}{3}\sqrt6 \Rightarrow \alpha=360^\circ\cdot\frac{\sqrt6}{3}\approx\boxed{294^\circ}$$

5. Ispitajte tok i nacrtajte graf funkcije
$$f(x)=\frac{3x^2-x}{x+1}.$$

Rješenje.

1) $\mathcal{D}(f)=\mathbb{R}\setminus\{-1\}$

2) $N(f)=?$
$$3x^2-x=0 \Rightarrow \begin{matrix} x_1=0 \\ x_2=1/3 \end{matrix}$$

$N(f) = \{0, \frac{1}{3}\}$

3) asimptote:

vertikalne $\boxed{x=-1}$

horizontalne $\lim\limits_{x\to\infty} f(x) = \infty$ nema

kosa
$$k = \lim_{x\to\infty}\frac{f(x)}{x} = \lim_{x\to\infty}\frac{3x^2-x}{x^2+x} \overset{:x^2}{=} \lim_{x\to\infty}\frac{3-\frac{1}{x}}{1+\frac{1}{x}} = 3$$

$$l = \lim_{x\to\infty}(f(x)-3x) = \lim_{x\to\infty}\left(\frac{\cancel{3x^2}-x-\cancel{3x^2}-3x}{x+1}\right) = \lim_{x\to\infty}\frac{-4x}{x+1} \overset{:x}{=} \lim_{x\to\infty}\frac{-4}{1+\frac{1}{x}} = -4$$

$\Rightarrow \boxed{y = 3x-4}$ kosa

4) $$f(x) = \frac{3x^2+3x-4x}{x+1} = \frac{3x(x+1)}{x+1} - 4\cdot\frac{x}{x+1} = 3x - 4\cdot\frac{x+1-1}{x+1} = 3x-4+\frac{4}{x+1}$$

$$f'(x) = 3 - \frac{4}{(x+1)^2} = 0 \Rightarrow (x+1)^2 = \frac{4}{3} \Rightarrow x+1 = \pm\frac{2\sqrt{3}}{3}$$

$$x_{1/2} = -1 \pm \frac{2\sqrt{3}}{3} \qquad x_1 \approx -2.15 \quad x_2 \approx 0.15$$

$$f''(x) = \frac{8}{(x+1)^3} \qquad f''(x_1) = \frac{8}{(\cancel{-1}-\frac{2\sqrt{3}}{3}+\cancel{1})^3} < 0 \Rightarrow MAX\ (\approx -2.15, \approx -13.92)$$

$$f''(x_2) = \frac{8}{(\cancel{-1}+\frac{2\sqrt{3}}{3}+\cancel{1})^3} > 0 \Rightarrow MIN\ (\approx 0.15, \approx -0.07)$$

$f''(x) = 0$ nema točke infleksije

5) rast: $f'(x) > 0$

$$3 > \frac{4}{(x+1)^2} \Rightarrow (x+1)^2 > \frac{4}{3} \Rightarrow |x+1| > \frac{2\sqrt{3}}{3}$$

$x+1 > \frac{2\sqrt{3}}{3}$, $\quad x+1 < -\frac{2\sqrt{3}}{3}$

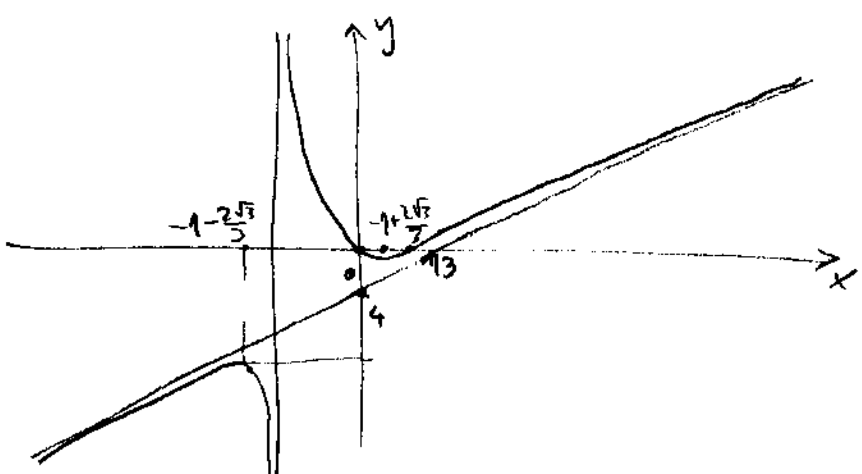
$x > -1+\frac{2\sqrt{3}}{3}$, $\quad x < -1-\frac{2\sqrt{3}}{3}$

	$\langle -\infty, -1-\frac{2\sqrt{3}}{3}\rangle$	$-1-\frac{2\sqrt{3}}{3}$	$\langle -\frac{2\sqrt{3}}{3}-1, 2\rangle$	2	$\langle 2, -1+\frac{2\sqrt{3}}{3}\rangle$	$-1+\frac{2\sqrt{3}}{3}$	$\langle -1+\frac{2\sqrt{3}}{3}, \infty\rangle$
f'	+	0	−	×	−	0	+
f	↗	MAX	↘	×	↘	MIN	↗

rast: $x \in \langle -\infty, -1-\frac{2\sqrt{3}}{3}\rangle \cup \langle -1+\frac{2\sqrt{3}}{3}, \infty\rangle$

pad: $x \in \langle -1-\frac{2\sqrt{3}}{3}, -1+\frac{2\sqrt{3}}{3}\rangle \setminus \{-1\}$

6) graf:



RJEŠENJA – MATEMATIKA 1, 5.7.2002.

1. Odredite $D(f)$, ako je $f(x)=\ln|\ln(15+2x-x^2)|$

Rješenje. • Zbog apsolutne vrijednosti već je zadovoljen uvjet da argument ln funkcije bude pozitivan, osim u slučaju kada je

$$\ln(15+2x-x^2)=0$$
$$15+2x-x^2=1$$
$$x^2-2x-14=0$$
$$x_{1,2}=\frac{2\pm\sqrt{4+56}}{2}=\frac{2\pm2\sqrt{15}}{2}=\boxed{1\pm\sqrt{15}\Rightarrow \text{ovo treba izbaciti}}$$

• uvjet na drugu logaritamsku funkciju:

$$15+2x-x^2>0$$
$$x^2-2x-15<0$$
$$x_{1,2}=\frac{2\pm\sqrt{4+60}}{2}=\frac{2\pm8}{2}=1\pm4$$
$$x_1=-3,\ x_2=5$$
$$\Rightarrow \boxed{x\in\langle-3,5\rangle}$$

$$\boxed{D(f)=\langle-3,5\rangle\setminus\{1\pm\sqrt{15}\}}\Leftarrow$$

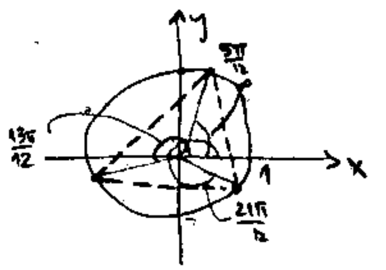
2. Riješite u $\mathbb{C}$: $\sqrt{2}x^3+1+i=0$. Slika!

Rješenje.
$$x^3=\frac{1-i}{\sqrt{2}}=-\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}$$
$$x_0=-\frac{\sqrt{2}}{2}-i\cdot\frac{\sqrt{2}}{2}$$
$$r=|x_0|=\sqrt{\tfrac{1}{2}+\tfrac{1}{2}}=1$$

$$\cos\varphi=-\frac{\sqrt{2}}{2}$$
$$\sin\varphi=-\frac{\sqrt{2}}{2}$$
$$\Rightarrow \varphi=\frac{5\pi}{4}$$

$$(x_0)_{0,1,2}=\cos\frac{\varphi+2k\pi}{3}+i\sin\frac{\varphi+2k\pi}{3}\qquad k=0,1,2$$
$$(x_0)_0=\cos\frac{\frac{5\pi}{4}}{3}+i\sin\frac{5\pi/4}{3}=\boxed{\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}}$$
$$(x_0)_1=\cos\frac{\frac{5\pi}{4}+2\pi}{3}+i\sin\frac{5\pi/4+2\pi}{3}=\boxed{\cos\frac{13\pi}{12}+i\sin\frac{13\pi}{12}}$$
$$(x_0)_2=\cos\frac{5\pi/4+4\pi}{3}+i\sin\frac{5\pi/4+4\pi}{3}=\boxed{\cos\frac{21\pi}{12}+i\sin\frac{21\pi}{12}}$$

Slika:



3. Izračunajte bez upotrebe L'Hospitalovog pravila limes $\lim_{x\to0}(\sin x(\sin x+1)+\cos^2 x)^{1/x}$.

Rješenje.

$$\lim_{x\to0}\left(\sin x(\sin x+1)+\cos^2 x\right)^{1/x}=$$

$$= e^{\ln \lim_{x\to0}\left(\sin x(\sin x+1)+\cos^2 x\right)^{1/x}}=$$

$$= e^{\lim_{x\to0}\ln\left(\sin^2 x+\sin x+\cos^2 x\right)^{1/x}}=$$

$$=1$$

$$= e^{\lim_{x\to0}\frac{1}{x}\ln(1+\sin x)} = e^{\lim_{x\to0}\frac{\ln(1+\sin x)}{\sin x}\cdot\frac{\sin x}{x}} =$$

proširimo sa $\sin x$

$$= e^{\lim_{x\to0}\frac{\ln(1+\sin x)}{\sin x}\cdot\overbrace{\boxed{\lim_{x\to0}\frac{\sin x}{x}}}^{=1}} = e^{\lim_{x\to0}\frac{\ln(1+\sin x)}{\sin x}} =$$

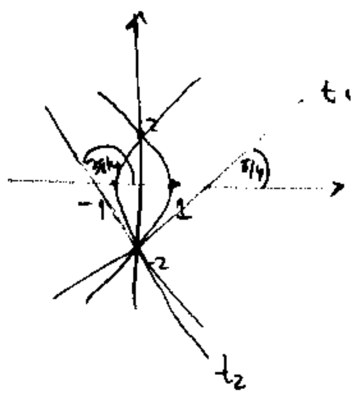
$$= e^{\lim_{\sin x\to0}\frac{\ln(1+\sin x)}{\sin x}} = e^{\overbrace{\boxed{\lim_{t\to0}\frac{\ln(1+t)}{t}}}^{=1}} = \boxed{e}$$

4. Nađite kut između krivulja: $y^2=4-4x$, $y^2=4+4x$. Odredite presječne točke, tangente i nacrtajte skicu.

Rješenje. tražimo presječne točke: $4-4x=4+4x$

$\Rightarrow x=0$

$\Rightarrow y^2=4 \Rightarrow y_{1,2}=\pm2$



To su točke $\boxed{(0,-2) \text{ i } (0,2)}$

Za $y=-2$ imamo $y_1=-\sqrt{4-4x}$

$y_2=-\sqrt{4+4x}$

$$y_1' = -\frac{1}{2\sqrt{4-4x}}\cdot(-4) = \frac{1}{\sqrt{1-x}}$$

$$y_1'(0)=1 \Rightarrow \underline{\operatorname{tg}\alpha_1=1} \Rightarrow \alpha_1=\frac{\pi}{4}$$

$$y_2' = -\frac{1}{2\sqrt{4+4x}}\cdot 4 = \frac{-1}{\sqrt{1+x}}$$

$$y_2'(0)=-1 \Rightarrow \underline{\operatorname{tg}\alpha_2=-1} \Rightarrow \alpha_2=-\frac{\pi}{4}$$

$\Rightarrow$ kut između tangenata je $\boxed{\pi/2}$

Postupak za točku $(0,2)$ je analogan.

5. Ispitajte tok i nacrtajte graf funkcije $f(x) = e^{\frac{-1}{x^2}}$.

Rješenje.

1) $\mathcal{D}(f) = \mathbb{R} \setminus \langle 0 \rangle$

2) $N(f) = ?$ nema nultočaka $\Rightarrow$ $N(f) = \emptyset$

3) V.A. $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} e^{-\frac{1}{x^2}} = e^{\lim\limits_{x\to 0} \frac{-1}{x^2}} = 0 \Rightarrow$ $\boxed{\lim\limits_{x\to 0} f(x) = 1}$!

H.A. $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} e^{-\frac{1}{x^2}} = 1$ (vidi graf!)

$\boxed{y = 1}$

4) $f'(x) = e^{-\frac{1}{x^2}} \cdot \frac{2}{x^3} = 0 \Rightarrow$ nema kandidata za ekstrem

5) $f''(x) = e^{-\frac{1}{x^2}} \cdot \frac{2}{x^3} \cdot \frac{2}{x^3} + e^{-\frac{1}{x^2}} \cdot \frac{-6}{x^4} = 0$

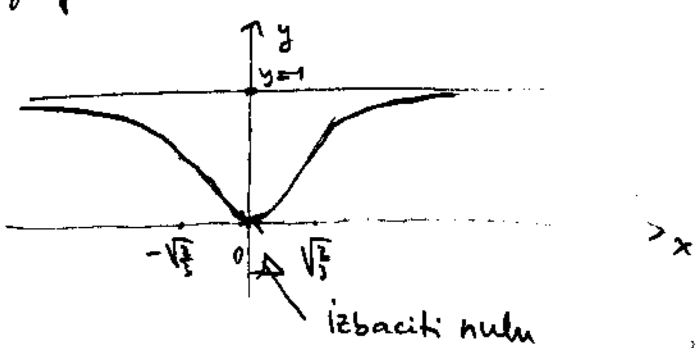
$\frac{2}{x^4} e^{-\frac{1}{x^2}} \left(\frac{2}{x^2} - 3 \right) = 0 \Rightarrow \frac{2}{3} = x^2 \Rightarrow x_{1,2} = \pm\sqrt{\frac{2}{3}} \approx \pm 0.81$

6) rast: $f'(x) > 0 \Rightarrow \frac{2}{x^3} > 0 \Rightarrow$ $\boxed{x > 0}$

pad: $\boxed{x < 0}$

	$\langle -\infty, 0 \rangle$	$x=0$	$\langle 0, \infty \rangle$
f'	$-$		$+$
f	$\searrow$		$\nearrow$

7) graf



>> MATEMATIKA I − 11.9.2002. − RJEŠENJA ZADATAKA

1. Riješite u $\mathbb{C}$ jednadžbu: $64z^2 - z^8 = 0$

<u>Rješenje:</u> $z^2(64 - z^6) = 0$

1) $z^6 = 64 = z_0 \quad n = 6 \,/\, |z_0| = 64 \,/\, \varphi = 0 \,/\, k = 0 \ldots 5$

$$z_k = \sqrt[n]{|z_0|}\left(\cos\frac{\varphi + 2k\pi}{n} + i \cdot \sin\frac{\varphi + 2k\pi}{n}\right)$$

$$z_k = \sqrt[6]{64}\left(\cos\frac{0 + 2k\pi}{6} + i \cdot \sin\frac{0 + 2k\pi}{6}\right) \Rightarrow z_k = 2\left(\cos\frac{k\pi}{3} + i\sin\frac{k\pi}{3}\right)$$

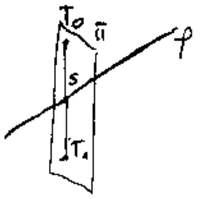
$$\Rightarrow \boxed{z_0 = 2,\ z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 1 + i\sqrt{3},\ z_2 = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = -1 + i\sqrt{3} \quad z_3 = -2\ \ z_4 = -1 - i\sqrt{3},\ z_5 = 1 - i\sqrt{3}}$$

2) $z^2 = 0 \Rightarrow \boxed{z_6 = z_7 = 0}$

---

2. Odredite točku simetričnu ishodištu s obzirom na pravac dan jednadžbom $\frac{x-2}{1} = \frac{y}{2} = \frac{z+3}{-3}$.

<u>Rješenje.</u> Ideja: nađi ravninu π kroz točku $T(0,0,0)$ koja je okomita na zadani pravac p. Presjek $\pi \cap p$ je točka S sa svojstvom da je $2\overrightarrow{T_0S} = \overrightarrow{T_0T_1}$, gdje je T_1 tražena točka.



$p \ldots \frac{x-2}{1} = \frac{y}{2} = \frac{z+3}{-3} \Rightarrow \vec{s}_p = \vec{i} + 2\vec{j} - 3\vec{k}$

$\pi \ldots ?\quad T_0(0,0,0),\ \ \vec{n}_\pi = \vec{s}_p = \vec{i} + 2\vec{j} - 3\vec{k}$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \Rightarrow \underline{x + 2y - 3z = 0}\ \ \pi$

$S = \pi \cap p \ldots x = t + 2,\ y = 2t,\ z = -3t - 3 \Rightarrow$

$t + 2 + 2 \cdot 2t - 3(-3t - 3) = 0$

$14t = -11 \Rightarrow t = -\frac{11}{14}$

$$S = \left(\frac{17}{14}, \frac{-22}{14}, \frac{-9}{14}\right) \Leftarrow \begin{cases} x_S = -\frac{11}{14} + 2 = \frac{17}{14} \\ y_S = -\frac{22}{14} \\ z_S = \frac{-9}{14} \end{cases}$$

$T_1 = (x_1, y_1, z_1) = ?\quad \overrightarrow{T_0T_1} = 2\overrightarrow{T_0S}$

$$\Rightarrow (x_1 - 0)\vec{i} + (y_1 - 0)\vec{j} + (z_1 - 0)\vec{k} = 2\left[\left(\frac{17}{14} - 0\right)\vec{i} + \left(\frac{-22}{14} - 0\right)\vec{j} + \left(\frac{-9}{14} - 0\right)\vec{k}\right]$$

$$\Rightarrow x_1 = \frac{17}{7},\ y_1 = -\frac{22}{7},\ z_1 = \frac{-9}{7} \Rightarrow \boxed{T_1 = \left(\frac{17}{7}, -\frac{22}{7}, \frac{-9}{7}\right)}$$

---

3. Odredite domenu funkcije $f(x) = \log[1 - \log_{3x}(3 - x)]$.

<u>Rješenje.</u> uvjeti:

$$\left.\begin{array}{l} 3x > 0 \Rightarrow x > 0 \\ 3x \neq 1 \Rightarrow x \neq 1/3 \\ 3 - x > 0 \Rightarrow x < 3 \end{array}\right\} \Rightarrow \underline{x \in \langle 0, 3\rangle \setminus \{\tfrac{1}{3}\}}\ \ (*)$$

$1 - \log_{3x}(3 - x) > 0 \Rightarrow$

$\log_{3x}(3-x) < 1$

a) $3x \in \langle 0,1\rangle$

$\boxed{x \in \langle 0, 1/3\rangle}$

$3-x > 3x$

$3 > 4x \Rightarrow \boxed{x < 3/4}$

$\Rightarrow \boxed{x \in \langle 0, \frac{1}{3}\rangle}$

b) $3x \in \langle 1, \infty\rangle$

$\boxed{x \in \langle 1/3, \infty\rangle}$

$3-x < 3x$

$3 < 4x \Rightarrow \boxed{x > 3/4}$

$\Rightarrow \boxed{x \in \langle 3/4, \infty\rangle}$

$\Rightarrow x \in \langle 0, \frac{1}{3}\rangle \cup \langle 3/4, \infty\rangle$, što zajedno s (*) daje

$$\boxed{\mathcal{D}(f) = \langle 0, \tfrac{1}{3}\rangle \cup \langle 3/4, 3\rangle}$$

---

4. Izračunajte bez L'Hospitalovog pravila $\lim\limits_{x\to 1} x^{\frac{x^2-\sqrt{x}}{\sqrt{x}-1}}$.

Rješenje.

$$\lim_{x\to 1} x^{\frac{x^2-\sqrt{x}}{\sqrt{x}-1}} = \lim_{x\to 1} e^{\ln x^{\frac{x^2-\sqrt{x}}{\sqrt{x}-1}}} = e^{\lim\limits_{x\to 1} \ln x^{\frac{x^2-\sqrt{x}}{\sqrt{x}-1}}} =$$

$$= e^{\lim\limits_{x\to 1} \frac{x^2-\sqrt{x}}{\sqrt{x}-1}\cdot \ln x} = e^{\lim\limits_{x\to 1} \frac{x^2-\sqrt{x}}{\sqrt{x}-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}\cdot \ln x} =$$

$$\boxed{\lim_{t\to 0} \frac{\ln(t+1)}{t}} = 1$$

$$= e^{\lim\limits_{x\to 1} \frac{(x^2-\sqrt{x})(\sqrt{x}+1)}{x-1}\cdot \ln x} = e^{\lim\limits_{x\to 1} \underbrace{(x^2-\sqrt{x})}_{=0}\cdot\underbrace{(\sqrt{x}+1)}_{2}\cdot \boxed{\lim\limits_{x\to 1}\frac{\ln x}{x-1}}} =$$

$$= e^{0\cdot 2\cdot 1} = e^0 = \boxed{1}$$

---

5. Odredite kvalitativni graf funkcije $f(x) = (x+1)\cdot \ln^2(x+1)$.

Rješenje 1) $\boxed{\mathcal{D}(f) = \langle -1, \infty\rangle}$

2) $N(f) = ?$ $\quad x+1=0 \Rightarrow x=-1$ nije nultočka, jer ne pripada domeni

$\ln^2(x+1) = 0 \Rightarrow x+1 = 1 \Rightarrow x = 0 \Rightarrow \boxed{N(f) = \{0\}}$

3) V.A. nema ($\lim\limits_{x\to -1_+} f(x) =$ (2 puta primijeniti L'Hospitalovo pravilo) $= 0$)

H.A. $\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} (x+1)\cdot \ln^2(x+1) = +\infty$ nema

K.A. $\lim\limits_{x\to +\infty} \frac{f(x)}{x} = \lim\limits_{x\to +\infty} \underbrace{\left(1+\frac{1}{x}\right)}_{1}\cdot \underbrace{\ln^2(x+1)}_{+\infty} = +\infty$ nema

4) $f'(x) = 1\cdot \ln^2(x+1) + \cancel{(x+1)}\cdot 2\ln(x+1)\cdot \frac{1}{\cancel{x+1}} = \ln(x+1)\cdot[\ln(x+1)+2]$

$f'(x) = 0 \Rightarrow \ln(x+1) = 0 \Rightarrow x+1 = 1 \Rightarrow \boxed{x=0}$

ili $\ln(x+1)+2 = 0 \Rightarrow x+1 = e^{-2} \Rightarrow \boxed{x = e^{-2}-1}$

stacionarne točke

5) $f''(x) = 2\ln(x+1)\cdot\frac{1}{x+1} + \frac{2}{x+1} = \frac{2}{x+1}\left(\ln(x+1)+1\right)$

$f''(0) = 2(1+\ln 1) = 2 > 0 \Rightarrow \quad (0, f(0)) = \boxed{(0,0)\ \text{MIN}}$

$f''(e^{-2}-1) = \frac{2}{e^{-2}\cancel{-1}\cancel{+1}}\left(\ln(e^{-2}\cancel{-1}\cancel{+1})+1\right) = 2e^{2}\cdot(-1) = -2e^{2} < 0 \Rightarrow$

$\Rightarrow \quad (e^{-2}-1, f(e^{-2}-1)) = \boxed{(e^{-2}-1, 4e^{-2})\ \text{MAX}}$

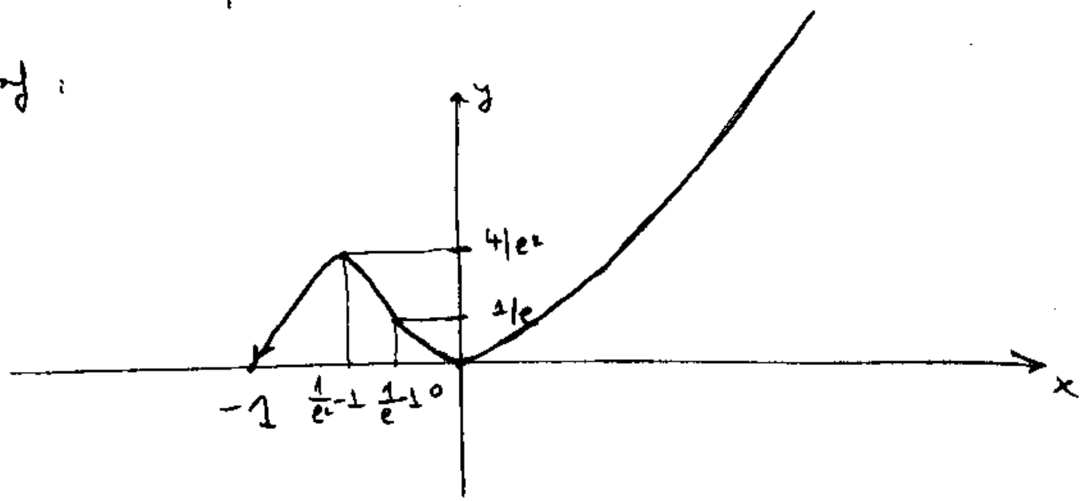
točke infleksije: $f''(x) = 0 \Rightarrow \ln(x+1)+1 = 0 \Rightarrow x+1 = e^{-1} \Rightarrow \quad \underline{x = e^{-1}-1}$

$(e^{-1}-1, f(e^{-1}-1)) = \boxed{(e^{-1}-1, e^{-1})}$ TOČKA INFLEKSIJE

6) rast-pad i tok:

	$\langle -1, e^{-2}-1\rangle$	$e^{-2}-1$	$\langle e^{-2}-1, 0\rangle$	0	$\langle e^{-2}-1, \infty\rangle$
f'	+	0	−	0	+
f	↗	MAX	↘	MIN	↗

7) graf:



MATEMATIKA 1 25.9.2002.

RJEŠENJA ZADATAKA

1. Nađite jednadžbu ravnine koja je okomita na ravninu $\Pi \ldots 2x+y-3z=4$, a sadrži pravac $p \ldots \begin{cases} x-y+z=2 \\ 3x-y+2z=4 \end{cases}$ (*).

Rješenje. Dovoljno je naći jednu točku T i vektor normale $\vec{n}$. Označimo s $\vec{n}_1$ vektor normale ravnine Π. Kako je ravnina koju tražimo okomita na Π, mora biti $\vec{n} \perp \vec{n}_1$.

S druge strane, $\vec{n} \perp \vec{s}$, gdje je $\vec{s}$ vektor smjera pravca p. S obzirom da je p dan kao presjek ravnina, mora biti $\vec{s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 3 & -1 & 2 \end{vmatrix} = -\vec{i}+\vec{j}+2\vec{k}$.

$$\vec{n} \perp \vec{n}_1,\ \vec{n} \perp \vec{s} \Rightarrow \vec{n} = \vec{n}_1 \times \vec{s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -3 \\ -1 & 1 & 2 \end{vmatrix} = \boxed{5\vec{i}-\vec{j}+3\vec{k}}$$

Još treba naći T: možemo uzeti bilo koju točku s pravca p, tj. bilo koju točku koja zadovoljava sustav (*). Uzmimo $\boxed{z=0} \Rightarrow \begin{array}{l} x-y=2 \\ 3x-y=4 \end{array} \Big/ - \Rightarrow 2x=2 \Rightarrow \boxed{x=1}$, $\boxed{y=-1}$

$\Rightarrow T=(1,-1,0)$

$\Rightarrow$ jednadžba ravnine je $5(x-1)-(y+1)+3(z-0)=0$

$$\Rightarrow \boxed{5x-y+3z=6}$$

---

2. Izračunajte $\lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$ bez upotrebe L'Hospitalovog pravila.

Rješenje.

$$\lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} \cdot \frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}} = \lim_{x\to 0} \frac{\cancel{1}+\sin x-\cancel{1}+\sin x}{x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)} =$$

$$= 2 \cdot \underbrace{\boxed{\lim_{x\to 0} \frac{\sin x}{x}}}_{=1} \cdot \boxed{\lim_{x\to 0} \frac{1}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}} = 2 \cdot 1 \cdot \frac{1}{2} = \boxed{1}$$

$$= \frac{1}{\sqrt{1+\sin 0}+\sqrt{1-\sin 0}} = \frac{1}{2}$$

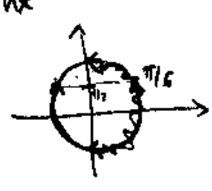
---

3. Odredite domenu funkcije $f(x) = \ln \dfrac{2\cos x}{1-2\sin x} + \sqrt{16-x^2}$.

Rješenje. Zbog $\sqrt{\ }$ mora biti $16-x^2 \geq 0 \Rightarrow x^2 \leq 16 \Rightarrow |x| \leq 4$, tj. $\boxed{x \in [-4,4]}$

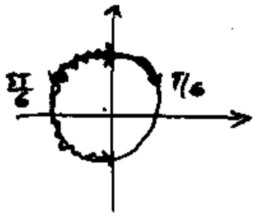
Zbog $\ln$ mora biti $\dfrac{2\cos x}{1-2\sin x} > 0$, tj. imamo dvije mogućnosti:

1) $\cos x > 0$
$1-2\sin x > 0 \rightarrow \sin x < 1/2$



$$\Rightarrow \boxed{x \in \bigcup_{k\in\mathbb{Z}} \left\langle 2k\pi - \frac{\pi}{2}, 2k\pi + \frac{\pi}{6} \right\rangle}$$

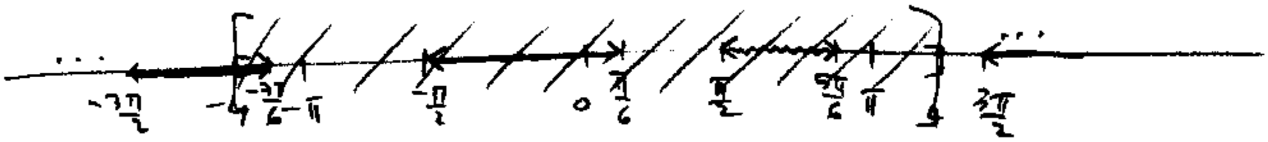
2) $\dfrac{\cos x < 0}{1-2\sin x < 0} \Rightarrow \sin x > 1/2$



$\Rightarrow \boxed{x \in \bigcup_{k\in\mathbb{Z}} \langle 2k\pi + \frac{\pi}{2}, 2k\pi + \frac{5\pi}{6} \rangle}$

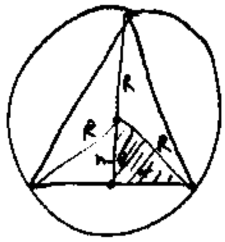
1) ∪ 2): $x \in \left(\bigcup_{k\in\mathbb{Z}} \langle 2k\pi - \frac{\pi}{2}, 2k\pi + \frac{\pi}{6} \rangle\right) \cup \left(\bigcup_{k\in\mathbb{Z}} \langle 2k\pi + \frac{\pi}{2}, 2k\pi + \frac{5\pi}{6} \rangle\right)$

Gledamo presjek ovog skupa sa segmentom $[-4,4]$:



$\Rightarrow \boxed{\mathcal{D}(f) = [-4, -\frac{7\pi}{6} \rangle \cup \langle -\frac{\pi}{2}, \frac{\pi}{6} \rangle \cup \langle \frac{\pi}{2}, \frac{5\pi}{6} \rangle}$

4.



$V = \frac{1}{3} r^2 \pi \cdot h$

$r^2 + (h-R)^2 = R^2 \Rightarrow r^2 + h^2 - 2hR = 0 \Rightarrow r^2 = 2hR - h^2$

$V = \frac{1}{3}\pi \underbrace{h(2hR - h^2)}_{f(h)}$

$f(h) = 2h^2R - h^3$

$f'(h) = 4hR - 3h^2 = 0 \Rightarrow (h \neq 0) \quad 4R = 3h \Rightarrow \boxed{h = \frac{4}{3}R}$

$r^2 = 2 \cdot \frac{4}{3} R \cdot R - \frac{16}{9} R^2 = \frac{8}{9} R^2 \Rightarrow \qquad \boxed{r = \frac{2\sqrt{2}}{3} R}$

$V_{MAX} = \frac{1}{3} \cdot \frac{8}{9} R^2 \cdot \pi \cdot \frac{4}{3} R \Rightarrow \boxed{V_{MAX} = \frac{32}{81} R^3 \pi}$

5. Ispitajte tok i nacrtajte graf funkcije $f(x) = \dfrac{2x-1}{(x-1)^2}$

Rješenje. 1) $\mathcal{D}(f) = \mathbb{R} \setminus \{1\}$

$N(f) = \{1/2\}$

2) V.A. $x = 1$

H.A. $\lim_{x\to+\infty} f(x) = 0$, isto i za $x \to -\infty \Rightarrow$ $y = 0$ H.A.

K.A. nema

3) $f'(x) = \dots = \dfrac{-2x}{(x-1)^3} = 0 \Rightarrow \boxed{x = 0}$ stacionarna točka

4) $f''(x) = \dots = \dfrac{4x+2}{(x-1)^4} = 0 \Rightarrow \boxed{x = -\frac{1}{2}}$ točka infleksije $\boxed{(-\frac{1}{2}, -\frac{8}{9})}$

$f''(0) = 2 > 0 \Rightarrow$ $\boxed{(0, f(0))\ \text{lokalni minimum}}$ $\boxed{(0,-1)}$

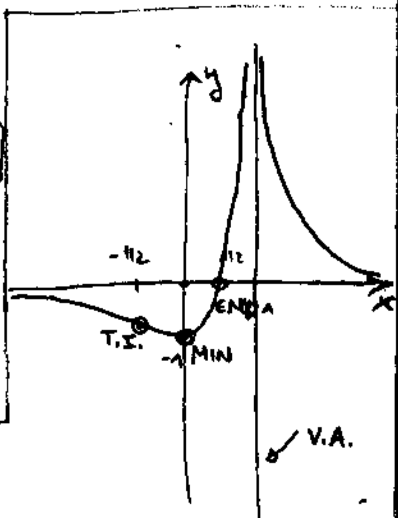
5) rast-pad: $f'(x) > 0 \Rightarrow \dfrac{x}{(x-1)^3} > 0 \Rightarrow x \in \langle 0, 1\rangle$ rast

$\Rightarrow x \in \langle -\infty, 0\rangle \cup \langle 1, \infty\rangle$ pad

6) t.t.:

	$\langle -\infty, 0\rangle$	0	$\langle 0, 1\rangle$	1	$\langle 1, \infty\rangle$
f'	−	0	+		−
f	↘	MIN	↗		↘

$\Rightarrow$ 7) graf:



MATEMATIKA 1 – 1. LISTOPADA 2007. – RJEŠENJA

Zadatak 1. Nađite domenu funkcije $f(x)=\ln(|x|+x)+\arccos\frac{2x-4}{x-1}$.

Rješenje a) $|x|+x>0$ zbog ln

b) $\frac{2x-4}{x-1}\in[-1,1]$ zbog arccos

a) $x\leq 0 \Rightarrow -x+x>0$ nema rješenja

$x>0 \Rightarrow x+x>0 \Rightarrow x>0 \Rightarrow \boxed{x\in\langle 0,\infty\rangle}$

b) I) $\frac{2x-4}{x-1}\geq -1 \Rightarrow \frac{2x-4}{x-1}+1\geq 0 \Rightarrow \frac{2x-4+x-1}{x-1}\geq 0 \Rightarrow \frac{3x-5}{x-1}\geq 0$

kako je $x>0$ (iz a)), onda svakako $x-1>0 \Rightarrow 3x-5\geq 0 \Rightarrow \boxed{x\geq\frac{5}{3}}$

II) $\frac{2x-4}{x-1}\leq 1 \Rightarrow \frac{2x-4}{x-1}-1\leq 0 \Rightarrow \frac{2x-4-x+1}{x-1}\leq 0 \Rightarrow \frac{x-3}{x-1}\leq 0$

kako je $x>0$ (iz (a)), slijedi je $x-3\leq 0 \Rightarrow \boxed{x\leq 3}$

I) ∩ II) daje $x\in[\frac{5}{3},3] \Rightarrow \boxed{\mathcal{D}(f)=[\frac{5}{3},3]}$

Zadatak 2. Izračunajte $\lim\limits_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$.

Rješenje.
$$\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)=\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\cdot\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}\right)$$

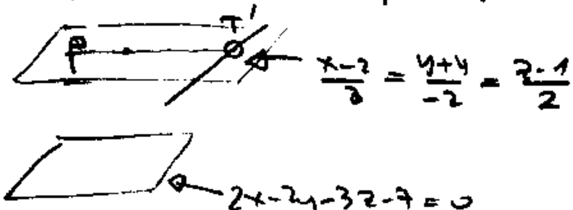
$$=\lim_{x\to\infty}\frac{\cancel{x}+\sqrt{x+\sqrt{x}}-\cancel{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}\overset{L:\sqrt{x}}{=}\lim_{x\to\infty}\frac{\sqrt{1+\frac{\sqrt{x}}{x}}}{\sqrt{1+\frac{\sqrt{x+\sqrt{x}}}{x}}+1}=$$

$$=\lim_{x\to\infty}\frac{\sqrt{1+\cancelto{0}{\frac{1}{\sqrt{x}}}}}{\sqrt{1+\sqrt{\underbrace{\tfrac{1}{x}}_{0}+\underbrace{\tfrac{1}{x\sqrt{x}}}_{0}}}+1}=\boxed{\frac{1}{2}}$$

Zadatak 3.

Odredite jednadžbu pravca koji prolazi točkom $T(3,-2,-4)$, usporedan je ravnini $3x-2y-3z-7=0$ i siječe pravac $\frac{x-2}{3}=\frac{y+4}{-2}=\frac{z-1}{2}$.

Rješenje.



Tražimo točku T' – ona je na presjeku ravnine paralelne sa zadanom (isti vektor normale!), a prolazi točkom T:

$$2(x-3)-2(y+2)-3(z+4)=0$$

$$2x-2y-3z-22=0$$

Tražimo presjekte ravnine sa zadanim pravcem: $x=3t+2,\ y=-2t-4,\ z=2t+1$

$$2(3t+2)-2(-2t-4)-3(2t+1)-22=0$$

$$4t=13 \Rightarrow t=\frac{13}{4} \Rightarrow \quad x=\frac{47}{4},\ y=\frac{-21}{2},\ z=\frac{15}{2} \Rightarrow \boxed{T'=\left(\frac{47}{4}, \frac{-21}{2}, \frac{15}{2}\right)}$$

Označimo pravac kojeg tražimo s p. Vrijedi $\vec{s}_p=\overrightarrow{TT'}=\frac{35}{4}\vec{i}-\frac{17}{2}\vec{j}+\frac{23}{2}\vec{k}$

Možemo uzeti $\vec{s}_p=35\vec{i}-34\vec{j}+46\vec{k}$, pa je $\boxed{p\ldots\ \frac{x-3}{35}=\frac{y+2}{-34}=\frac{z+4}{46}}$

<u>Zadatak</u> 4. Broj 36 rastavite na dva pozitivna faktora tako da zbroj njihovih kvadrata bude minimalan.

<u>Rješenje.</u>

$$36=x\cdot y \Rightarrow y=\frac{36}{x}$$

$$f(x,y)=x^2+y^2$$

$$\tilde{f}(x)=x^2+\frac{36^2}{x^2} \leftarrow \text{tražimo minimum ove funkcije}$$

$$\tilde{f}'(x)=2x-\frac{36^2}{x^3}\cdot 2=0 \Rightarrow \quad x^4=36^2 \ /\ \sqrt[4]{\ }$$

$$x=\sqrt{36}=6$$

$$y=6$$

Radi se o brojevima 6 i 6.

<u>Zadatak 5.</u> Nacrtajte graf funkcije $f(x)=1-e^{-\cos x}$ (funkcija je periodična!).

<u>Rješenje.</u> 1) $\mathcal{D}(f)=\mathbb{R}$

$N(f)=?\quad f(x)=0 \Rightarrow \quad e^{-\cos x}=1=e^0 \Rightarrow \quad \cos x=0 \Rightarrow \boxed{x\in\{k\pi+\frac{\pi}{2} \mid k\in\mathbb{Z}\}}$

$N(f)=\{k\pi+\frac{\pi}{2} \mid k\in\mathbb{Z}\}$

2) Asimptote: nema vertikalnih asimptota

horizontalne: $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\left(1-\frac{1}{e^{\boxed{\cos x}}}\right)$, što ne konvergira, jer funkcija

$\cos$ ne konvergira $\Rightarrow$ nema horizontalnih asimptota

Iz istog razloga nema ni kosih asimptota

3) $f'(x)=-e^{-\cos x}\cdot(-\cos x)'=e^{-\cos x}\cdot(-\sin x)=0 \Rightarrow \quad \sin x=0$

$\Rightarrow \boxed{x\in\{k\pi \mid k\in\mathbb{Z}\}}$

stacionarne točke

4) $f''(x)=e^{-\cos x}\cdot(-\cos x)'\cdot(-\sin x)+e^{-\cos x}\cdot(-\sin x)'=$

$=\underbrace{-\sin^2 x}_{\cos^2 x-1}\, e^{-\cos x}+e^{-\cos x}\cdot(-\cos x)=e^{-\cos x}(\cos^2 x-\cos x-1)$

$f''(x)=0 \Rightarrow \cos^2 x-\cos x-1=0 \quad \cos x=t \Rightarrow t^2-t-1=0$

$t_{1,2}=\frac{1\pm\sqrt{1+4}}{2}=\frac{1\pm\sqrt{5}}{2} \Rightarrow \cos x=\frac{1-\sqrt{5}}{2}$ ili $\cos x=\frac{1+\sqrt{5}}{2}>1$ nemoguće

$\boxed{x=\arccos\frac{1-\sqrt{5}}{2}}$ ← to su točke infleksije

Uvrštavamo u f'' stacionarne točke; razlikujemo „parne" i „neparne", tj. one oblika $2l\pi$ i $2l\pi+\pi$, $l\in\mathbb{Z}$:

$f''(2l\pi)=e^{-\cos 2l\pi}(\cos^2 2l\pi-\cos 2l\pi-1)=e^{-1}(\cancel{1}-\cancel{1}-1)=-\frac{1}{e}<0 \Rightarrow$ MAX

$f''(2l\pi+\pi)=e^{-\cos(2l\pi+\pi)}(\cos^2(2l\pi+\pi)-\cos(2l\pi+\pi)-1)=e^{1}(\cancel{1}-(-1)-\cancel{1})=e>0$ MIN

$f(2l\pi)=1-e^{-\overset{1}{\boxed{\cos 2l\pi}}}=1-\frac{1}{e} \Rightarrow$

$f(2l\pi+\pi)=1-e^{-\underset{-1}{\boxed{\cos(2l\pi+\pi)}}}=1-e \Rightarrow$

$(2l\pi, 1-\frac{1}{e})$ su točke maksimuma	$l\in\mathbb{Z}$
$(2l\pi+\pi, 1-e)$ su točke minimuma	

5) $f'(x)>0 \Rightarrow e^{-\cos x}>0\ \forall x \Rightarrow -\sin x>0 \Rightarrow \sin x<0 \Rightarrow$

$x\in\bigcup_{k\in\mathbb{Z}}\langle 2k\pi-\pi, 2k\pi\rangle$ rast
$x\in\bigcup\langle 2k\pi, 2k\pi+\pi\rangle$ pad

6) tok:

	...	-3π		-2π		$-\pi$		0		π		2π	...
f'	...	0	+	0	−	0	+	0	−	0	+	0	...
f	...	MIN	↗	MAX	↘	MIN	↗	MAX	↘	MIN	↗	MAX	...

7) graf (funkcija je očito periodička s periodom 2π)