MATEMATIKA 1

PISMENI ISPITI 2002. - RJEŠENJA

20. veljače
11. svibnja
18. lipnja
2. srpnja
9. srpnja
11. rujna
25. rujna
1. listopada

MATEMATIKA I – 20. VELJAČE 2002.

RJEŠENJA

1. Odredite domenu funkcije $f(x) = \ln|\ln(15+2x-x^2)| + \dfrac{1}{(\sqrt{\pi} - 2\sqrt{\arcsin(x-1)})}$.

Rješenje. ln: $|\ln(15+2x-x^2)| > 0 \Rightarrow \ln(15+2x-x^2) \neq 0$

$\Rightarrow 15+2x-x^2 \neq 1$

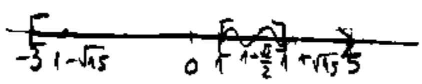
$\Rightarrow x^2-2x-14 \neq 0$

$D = 4+56 = 60$

$\Rightarrow x_{1,2} = \dfrac{2\pm 2\sqrt{15}}{2} = 1\pm\sqrt{15}$

$\Rightarrow \boxed{x \notin \{1-\sqrt{15}, 1+\sqrt{15}\}}$

osim toga, mora biti $15+2x-x^2 > 0$



$x^2-2x-15 < 0$, $D = 4+60 = 64$

$x_{1,2} = \frac{2\pm 8}{2} = 1\pm 4 \Rightarrow x_1 = -3$, $x_2 = 5$

$\Rightarrow \boxed{x \in \langle -3, 5\rangle}$

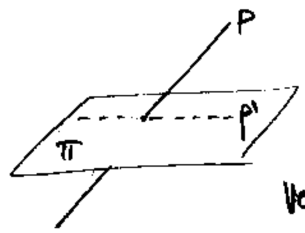
$\sqrt{\ }$: $\arcsin(x-1) \geq 0 \Rightarrow 1 \geq x-1 \geq 0 \Rightarrow 2 \geq x \geq 1 \Rightarrow \boxed{x \in [1,2]}$

razlomak: $\sqrt{\pi} \neq 2\sqrt{\arcsin(x-1)} \Rightarrow \frac{\pi}{4} \neq \arcsin(x-1)$ /sin $\Rightarrow \frac{\sqrt{2}}{2} \neq x-1 \Rightarrow \boxed{x \neq \frac{\sqrt{2}}{2}+1}$

$\Rightarrow \boxed{\mathcal{D}(f) = [1,2] \setminus \{1+\frac{\sqrt{2}}{2}\}}$

2. Odredite ortogonalnu projekciju pravca $p \ldots \begin{cases} x=1 \\ y=0 \end{cases}$ na ravninu $\pi \ldots x+y+z=2$.

Rješenje.



Ideja: provući kroz p ravninu π' koja je okomita na π; projekcija p' pravca p na π je presjek $\pi \cap \pi'$

Vektor smjera $\vec{s}$ pravca p dobije se kao vektorski produkt vektora normala ravnina $x=1$ i $y=0$: $\vec{s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \vec{k}$

Vektor normale $\vec{n}'$ ravnine π' dobijemo kao vektorski produkt vektora normale ravnine π i vektora smjera $\vec{s}$ pravca p: $\vec{n}' = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -\vec{i}+\vec{j}$

Vektor smjera $\vec{s}'$ projekcije p' dobijemo kao vektorski produkt vektora normale ravnine π i π': $\vec{s}' = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \vec{i}+\vec{j}-2\vec{k}$

Kao točku na p' možemo uzeti točku presjeka p i π: $x=1, y=0 \Rightarrow 1+0+z=2 \Rightarrow z=1$

Sada je jednadžba pravca p': $\boxed{\dfrac{x-1}{1} = \dfrac{y}{1} = \dfrac{z-1}{-2}}$

3. $$\lim_{x\to -2} \frac{\sqrt{x+3}-1}{\operatorname{tg}(\pi(x+3))} = \begin{bmatrix} \text{supst. } x+2=t \\ t\to 0\end{bmatrix} = \lim_{t\to0}\frac{\sqrt{t+1}-1}{\operatorname{tg}(\pi(t+1))} =$$

$$= \lim_{t\to0}\frac{\sqrt{t+1}-1}{\operatorname{tg}(\pi t+\pi)} = \lim_{t\to0}\frac{\sqrt{t+1}-1}{\sin(\pi t+\pi)}\cdot\cos(\pi t+\pi) = \begin{bmatrix}\sin(x+\pi)=-\sin x\\ \cos(x+\pi)=-\cos x\end{bmatrix} =$$

$$= \lim_{t\to0}\frac{\sqrt{t+1}-1}{\sin\pi t}\cdot\cos\pi t = \lim_{t\to0}\left(\frac{\sqrt{t+1}-1}{\pi t}\cdot\frac{\pi t}{\sin \pi t}\cdot\frac{\sqrt{t+1}+1}{\sqrt{t+1}+1}\cdot\cos\pi t\right)=$$

$$= \lim_{t\to0}\left(\frac{\cancel{t}+\cancel{1}-\cancel{1}}{\pi\cancel{t}(\sqrt{t+1}+1)}\cdot \underbrace{\cos(\pi t)}_{1}\right)\cdot \underbrace{\lim_{\substack{t\to0\\(\pi t\to 0)}}\cancel{\frac{\sin\pi t}{\sin\pi t}}}_{1} = \pi$$

$$= \frac{1}{2\pi}\cdot 1 = \boxed{\frac{1}{2\pi}}$$

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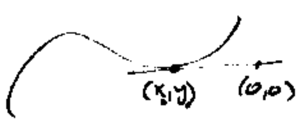
4. Odredite jednadžbu tangente na $C \dots y=x^4-x+3$ koja prolazi ishodištem koord. sustava.

Rješenje.



Tražimo (x,y) na krivulji C tako da je pravac kroz (x,y) i $(0,0)$ tangenta na C u točki (x,y).

Prema formuli za jednadžbu pravca kroz 2 točke, t je pravac $y=\frac{y_0}{x_0}x$. Kako je (x_0,y_0) točka s krivulje, to mora biti

$y_0 = x_0^4-x_0+3$.

S druge strane, koeficijent smjera tangente u točki (x_0,y_0) mora biti jednak $y'(x_0)$.
Dakle mora biti $\frac{y_0}{x_0} = 4x_0^3-1$, tj.

$$x_0^4-x_0+3 = x_0(4x_0^3-1)$$
$$x_0^4-\cancel{x_0}+3-4x_0^4+\cancel{x_0}=0$$
$$3=3x_0^4 \Rightarrow x_0^4=1 \Rightarrow x_0\in\{+1,-1\}$$

$x_0=-1 \Rightarrow y_0=1+1+3=5 \Rightarrow t_1 \dots \boxed{y=-5x}$

$x_0=1 \Rightarrow y_0=3 \Rightarrow t_2 \dots \boxed{y=3x}$

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5. Ispitajte tok i nacrtajte graf funkcije $f(x)=\frac{x^2}{\ln x^2}$.

Rješenje. 1. $D(f)=?$ Treba biti: 1) $x^2>0 \Rightarrow x\neq 0$

2) $\ln x^2\neq 0 \Rightarrow x^2\neq 1 \Rightarrow x\neq 1$, $x\neq -1$

$\Rightarrow \boxed{D(f)=\mathbb{R}\setminus\{0,-1,1\}}$

2. $N(f)=?$ $x^2=0 \Rightarrow x=0$ ne može biti jer $0\notin D(f) \Rightarrow \boxed{N(f)=\emptyset}$

3. Asimptote: V.A. $x=1, x=-1$; u $x=0$ nemamo $\lim_{x\to0} f(x)\to\pm\infty \to x=0$ nije V.A.

H.A. $\lim_{x\to\infty}\frac{x^2}{\ln x^2} \overset{L'H}{=} \lim_{x\to\infty}\frac{2\cancel{x}}{\frac{1}{x^2}\cdot 2\cancel{x}} = \lim_{x\to\infty} x^2 = \infty \Rightarrow$ nema

K.A. $k=\lim_{x\to\infty}\frac{x^{\cancel{2}}}{\cancel{x}\ln x^2} \overset{L'H}{=} \lim_{x\to\infty}\frac{1}{\frac{1}{x^2}\cdot 2x} = \lim_{x\to\infty}\frac{x^{\cancel{2}}}{2\cancel{x}} = \infty \Rightarrow$ nema

20.02.02

4. $f'(x) = \dfrac{2x\cdot\ln x^2 - \cancel{x^2}\cdot\frac{1}{\cancel{x^2}}\cdot 2x}{(\ln x^2)^2} = \dfrac{2x(\ln x^2 - 1)}{(\ln x^2)^2} = 0 \Rightarrow$ $x = 0$ ne može biti, jer $x \notin D(f)$

$\ln x^2 = 1 \Rightarrow x^2 = e$
$\Rightarrow \underline{x_1 = -\sqrt{e},\ x_2 = \sqrt{e}}$
kritične točke

5. $f''(x) = \dfrac{\left(2(\ln x^2 - 1) + 2\cancel{x}\left(\frac{1}{\cancel{x^2}}\cdot 2\cancel{x}\right)\right)\cdot(\ln x^2)^2 - 2\cancel{x}(\ln x^2 - 1)\cdot 2(\ln x^2)\cdot\frac{1}{\cancel{x^2}}\cdot 2\cancel{x}}{(\ln x^2)^4}$

$f''(x) = \dfrac{(2\ln x^2 - \cancel{2} + \cancel{2})(\ln x^2)^{\cancel{2}} - 8(\cancel{\ln x^2})(\ln x^2 - 1)}{(\ln x^2)^{\cancel{4}}}$

$f''(x) = \dfrac{2(\ln x^2)^2 - 8(\ln x^2 - 1)}{(\ln x^2)^3}$

$f''(\pm\sqrt{e}) = \dfrac{2(\ln e)^2 - 8(\ln e - 1)}{(\ln e)^3} = \dfrac{2 - 8\cdot(1-1)}{1^3} = 2 > 0 \Rightarrow$ u $\sqrt{e}$ i $-\sqrt{e}$ se postiže lok. minimum

$f(\pm\sqrt{e}) = \dfrac{e}{\ln e} = e \Rightarrow \boxed{(-\sqrt{e}, e),\ (\sqrt{e}, e)}$

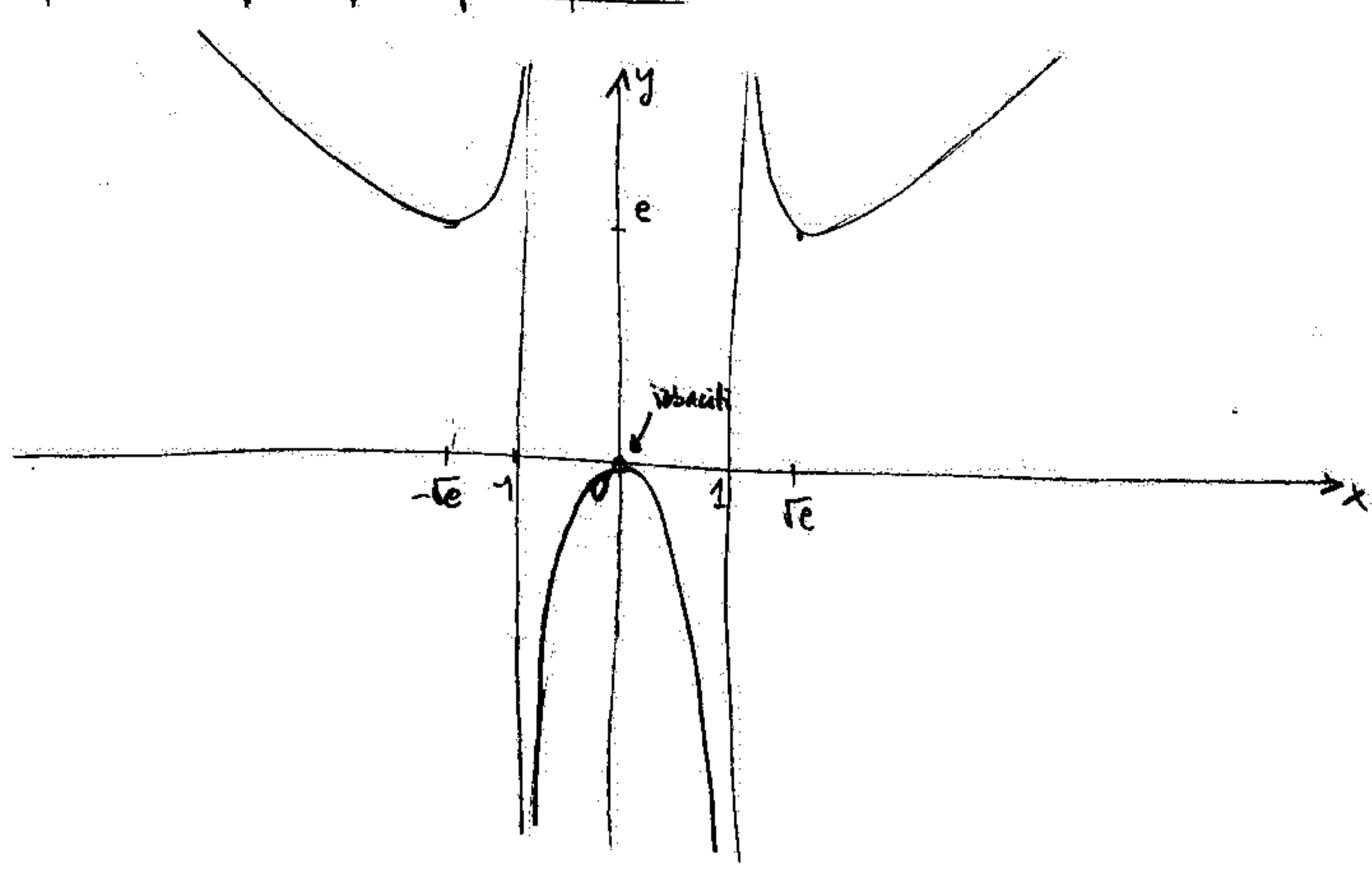
Točke infleksije: uz supstituciju $\ln x^2 = t$ rješavamo $2t^2 - 8t + 8 = 0$
$t^2 - 4t + 4 = 0$
$(t-2)^2 = 0 \Rightarrow t = 2$
$\ln x^2 = 2 \Rightarrow x^2 = e^2 \Rightarrow \underline{x_1 = -e}$, $\underline{x_2 = e}$

6. $f'(x) < 0 \Rightarrow x(\ln x^2 - 1) < 0$
Dobije se $\underline{x \in \langle -\infty, -\sqrt{e}\rangle \cup \langle 0, \sqrt{e}\rangle \setminus \{1\}}$ pad

$f'(x) > 0 \Rightarrow$ dobije se $\underline{x \in \langle -\sqrt{e}, 0\rangle \setminus \{-1\} \cup \langle \sqrt{e}, \infty\rangle}$ rast

$D(f)$	$\langle -\infty, -\sqrt{e}\rangle$	$-\sqrt{e}$	$\langle -\sqrt{e}, -1\rangle$	-1	$\langle -1, 0\rangle$	0	$\langle 0, 1\rangle$	1	$\langle 1, \sqrt{e}\rangle$	$\sqrt{e}$	$\langle \sqrt{e}, \infty\rangle$
f'	−	0	+		+		−		−	0	+
f	↘	MIN	↗		↗		↘		↘	MIN	↗

7.



MATEMATIKA 1.

11.5.2002.

RJEŠENJA ZADATAKA

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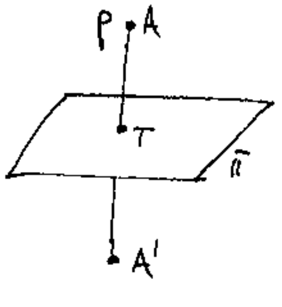
1. Odredite točku simetričnu točki $A(1,2,0)$ obzirom na ravninu $2x+3y-4z+21=0$.

$$\underset{x_A}{1},\ \underset{y_A}{2},\ \underset{z_A}{0}$$

Rješenje.

$A' = ?$



ideja je da kroz točku A povučemo pravac okomit na ravninu π – presjek tog pravca s ravninom je točka T, a onda je (uz $A'=(x_{A'},y_{A'},z_{A'})$, $T=(x_T,y_T,z_T)$)

$$\frac{x_A+x_{A'}}{2}=x_T,\quad \frac{y_A+y_{A'}}{2}=y_T,\quad \frac{z_A+z_{A'}}{2}=z_T \text{ (polovište)}$$

Ako označimo koeficijent smjera pravca p sa $\vec{s}$, očito je $\vec{s}=\vec{n}$, gdje je $\vec{n}$ vektor normale od π

$$\Rightarrow \vec{s}=2\vec{i}+3\vec{j}-4\vec{k},\ p\sim\vec{s} \qquad p\ldots\ \frac{x-1}{2}=\frac{y-2}{3}=\frac{z}{-4} \rightarrow \begin{aligned} x&=2t+1\\ y&=3t+2\\ z&=-4t\end{aligned}$$

Uvrstimo tu točku u jednadžbu ravnine (tražimo $T=p\cap\pi$):

$$2(2t+1)+3(3t+2)-4(-4t)+21=0 \Rightarrow 29t=-29 \Rightarrow t=-1 \Rightarrow x_T=-1,\ y_T=-1,\ z_T=4$$

$$\Rightarrow x_{A'}=2\cdot(-1)-1 \rightarrow \boxed{x_{A'}=-3}\ /\ y_{A'}=2\cdot(-1)-2 \Rightarrow \boxed{y_{A'}=-4}\ /\ z_{A'}=2\cdot4 \Rightarrow \boxed{z_{A'}=8}$$

$$\Rightarrow \boxed{A'=(-3,-4,8)}$$

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2. Izračunajte $\lim\limits_{x\to-\infty} f(x)+\lim\limits_{x\to\infty} f(x)$ bez upotrebe L'Hospitala ako je $f(x)=\sqrt{x^2+2x+3}-\sqrt{x^2-2x+3}$.

Rješenje.

$$f(x)=\underset{\substack{\uparrow\\ \text{antiracionalizacija}}}{=}\left(\sqrt{x^2+2x+3}-\sqrt{x^2-2x+3}\right)\cdot\frac{\sqrt{x^2+2x+3}+\sqrt{x^2-2x+3}}{\sqrt{x^2+2x+3}+\sqrt{x^2-2x+3}}=$$

$$=\frac{\left(\sqrt{x^2+2x+3}\right)^2-\left(\sqrt{x^2-2x+3}\right)^2}{\sqrt{x^2+2x+3}+\sqrt{x^2-2x+3}}=\frac{\cancel{x^2}+2x+\cancel{3}-\cancel{x^2}+2x-\cancel{3}}{\sqrt{x^2+2x+3}+\sqrt{x^2-2x+3}}=$$

$$=\frac{4x}{\sqrt{x^2}\sqrt{1+\frac{2}{x}+\frac{3}{x^2}}+\sqrt{x^2}\cdot\sqrt{1-\frac{2}{x}+\frac{3}{x^2}}}\underset{\substack{\uparrow\\ \text{u limesu}}}{=}\frac{4\cancel{x}\ \pm \text{ ovisno o } x\to-\infty \text{ ili } x\to+\infty}{|\cancel{x}|\left(\sqrt{1+\underbrace{\cancel{\tfrac{2}{x}}+\cancel{\tfrac{3}{x^2}}}_{0}}+\sqrt{1-\underbrace{\cancel{\tfrac{2}{x}}+\cancel{\tfrac{3}{x^2}}}_{0}}\right)}$$

$$\Rightarrow \lim_{x\to-\infty} f(x)+\lim_{x\to\infty} f(x)=-\frac{4}{1+1}+\frac{4}{1+1}=-2+2=\boxed{0}$$

3. Odredite domenu funkcije $f(x)=\ln\left(\frac{1}{(\sqrt{x-2}-1)(\sqrt{x-3}-2)}\right)$

Rješenje. Mora biti $\left.\begin{array}{l} x-2\geq 0 \\ x-3\geq 0\end{array}\right\} \Rightarrow \boxed{x\geq 3}_{(*)}$ (zbog korijena)

Zbog f-je ln je $\frac{1}{(\sqrt{x-2}-1)(\sqrt{x-3}-2)}>0$, tj.

$$(\sqrt{x-2}-1)(\sqrt{x-3}-2)>0$$

Imamo 2 mogućnosti: a) $\sqrt{x-2}-1>0$ $\sqrt{x-3}-2>0$ ili b) $\sqrt{x-2}-1<0$ $\sqrt{x-3}-2<0$

a) $\Rightarrow \sqrt{x-2}>1 \,/()^2$, $\sqrt{x-3}>2\,/()^2$ $\Rightarrow x-2>1 \Rightarrow \boxed{x>3}$, $x-3>4\Rightarrow \boxed{x>7}$ $\Rightarrow \boxed{x>7}$

b) $\sqrt{x-2}<1\,/()^2$, $\sqrt{x-3}<2\,/()^2$ $\Rightarrow x-2<1\Rightarrow\boxed{x<3}$, $x-3<4\Rightarrow\boxed{x<7}$ $\Rightarrow \boxed{x<3}$

(**) $\boxed{x\in\langle-\infty,3\rangle\cup\langle 7,\infty\rangle} \Leftarrow$

Kada pogledamo (*) i (**), vidimo da je $\boxed{\mathcal{D}(f)=\langle 7,\infty\rangle}$

4. Na krivulji $y=x^3-3x^2+3x-4$ povucite tangentu okomitu na pravac $x=2002$.

Rješenje. Pravci okomiti na pravac $x=2002$ su oni oblika $y=c$, $c\in\mathbb{R}$ konstanta
$\Rightarrow y=0\cdot x+c \Rightarrow \underline{\text{koef. smjera}=0}$

$y'(x)=3x^2-6x+3=0 \Rightarrow x^2-2x+1=0\Rightarrow \boxed{x=1}\Rightarrow y=1-3+3-4\Rightarrow \boxed{y=-3}$

$\Rightarrow \boxed{y=-3}$ je tražena tangenta

5. Ispitajte tok i nacrtajte graf f-je $f(x)=x\cdot(\ln x)^2$

Rješenje. 1) $\mathcal{D}(f)=\langle 0,\infty\rangle$ (zbog ln f-je)

2) $N(f)=?$ $\cancel{x=0}$ ili $\ln x=0$ $(0\notin \mathcal{D}(f)!)\Rightarrow x=1 \Rightarrow N(f)=\{1\}$

3) V.A. nema, jer $\lim\limits_{x\to 0_+} x(\ln x)^2=\lim\limits_{x\to 0_+}\frac{(\ln x)^2}{\frac{1}{x}}\overset{L'H}{=}\lim\limits_{x\to 0_+}\frac{2\ln x\cdot\frac{1}{\cancel{x}}}{-\frac{\cancel{1}}{x^2}}=-2\lim\limits_{x\to 0_+}\frac{\ln x}{\frac{1}{x}}\overset{L'H}{=}$

$$=-2\lim_{x\to 0_+}\frac{\frac{1}{x}}{-\frac{1}{x^2}}=2\lim_{x\to 0_+}x=0$$

K.A. nema

H.A. $\lim\limits_{x\to+\infty} f(x)=+\infty$, nema

4) $f'(x)=(\ln x)^2+\cancel{x}\cdot 2\ln x\cdot\frac{1}{\cancel{x}}=\ln x(\ln x+2)=0\Rightarrow \ln x=0$ ili $\ln x+2=0$
$\underline{x_1=1}$ $\underline{x_2=e^{-2}}$ kandidati za ekstrem

rast-pad: $\underline{\text{rast}}$ $f'(x)>0 \Rightarrow$ $\ln x>0$, $\ln x+2>0$ ili $\ln x<0$, $\ln x+2<0$

$\ln x>0\Rightarrow \boxed{x>1}$ ili $\ln x<-2\Rightarrow \boxed{x<\frac{1}{e^2}}$

$\Rightarrow$ rast na $\langle 0, \frac{1}{e^2}\rangle \cup \langle 1, \infty\rangle$

$\Rightarrow$ pad na $\langle \frac{1}{e^2}, 1\rangle$

5) $f''(x) = 2\ln x \cdot \frac{1}{x} + 2 \cdot \frac{1}{x} = \frac{2}{x}(\ln x + 1) = 0 \Rightarrow \ln x = -1 \Rightarrow \boxed{x = \frac{1}{e} \text{ T. infleksije}}$

$f''(1) = 2(\ln 1 + 1) = 2 > 0 \Rightarrow (1, f(1)) = \boxed{(1,0) \text{ MIN}}$

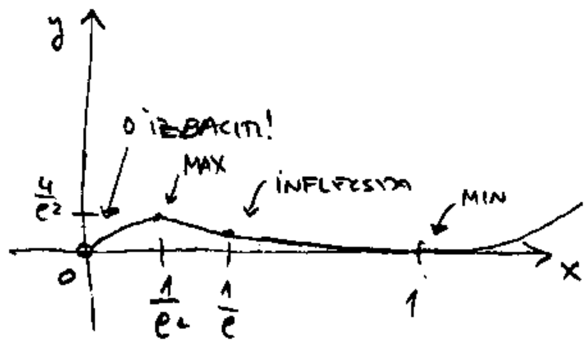
$f''(\frac{1}{e^2}) = 2e^2(-2+1) = -2e^2 < 0 \Rightarrow (\frac{1}{e^2}, f(\frac{1}{e^2})) = \boxed{(\frac{1}{e^2}, \frac{4}{e^2}) \text{ MAX}}$

6) tok:

$D(f)$	$\langle 0, \frac{1}{e^2}\rangle$	$\frac{1}{e^2}$	$\langle \frac{1}{e^2}, 1\rangle$	1	$\langle 1, \infty\rangle$
f'	+	0	−	0	+
f	↗	MAX	↘	MIN	↗

$\Rightarrow$

7) graf

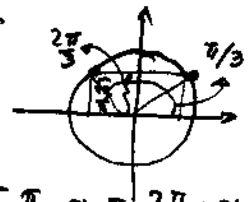


MATEMATIKA I / 18.6.2002. / RJEŠENJA

1. Odredite domenu funkcije $f(x)=\sqrt{\sin(\arccos x)-\frac{\sqrt{3}}{2}}$.

Rješenje. $\sin(\arccos x)-\frac{\sqrt{3}}{2}\geq 0$ i $\boxed{x\in[-1,1]}$ (zbog arccos)

$\sin(\arccos x)\geq\frac{\sqrt{3}}{2}$



$\Rightarrow$

$$\arccos x\in\bigcup_{k\in\mathbb{Z}}\left[\frac{\pi}{3}+2k\pi,\frac{2\pi}{3}+2k\pi\right]$$

Kako je skup vrijednosti funkcije arccos jednak $[0,\pi]$, to mora biti $k=0$, tj. $\arccos x\in\left[\frac{\pi}{3},\frac{2\pi}{3}\right]$

$\frac{\pi}{3}\leq\arccos x\leq\frac{2\pi}{3}\Rightarrow\boxed{x\in\left[-\frac{1}{2},\frac{1}{2}\right]}$

$\Rightarrow\boxed{\mathcal{D}(f)=\left[-\frac{1}{2},\frac{1}{2}\right]}$

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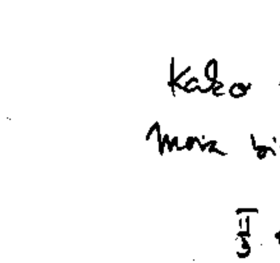
2. Odredite jednadžbu ravnine koja sadrži točke $T_1(1,2,0)$ i $T_2(2,3,1)$, a okomita je na ravninu $2x+3y-4z=0$.

Rješenje.



$\pi_1 \ldots\ 2x+3y-4z=0$

$\pi_2=?$

$\vec{n}_2\perp\vec{n}_1,\ \overrightarrow{T_1T_2}=\vec{i}+\vec{j}+\vec{k}$

$$\Rightarrow\ \vec{n}_2=\vec{n}_1\times\overrightarrow{T_1T_2}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\1&1&1\\2&3&-4\end{vmatrix}=-7\vec{i}+6\vec{j}+\vec{k}$$

Jednadžba ravnine s vektorom normale $\vec{n}_2$, a s točkom $T_1(1,2,0)$ glasi

$-7(x-1)+6(y-2)+1\cdot(z-0)=0\ /\cdot(-1)\Rightarrow\ \boxed{7x-6y-z+5=0}$

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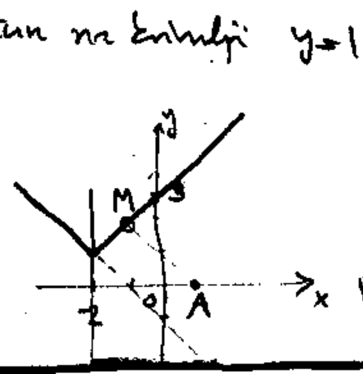
3. Koristeći diferencijalni račun na krivulji $y=|x+2|+1$ nađite točku najbližu točki $A(1,0)$.

Rješenje.

$x+2=0$

$x=-2$

kandidat točka



$x\leq-2$

$y=-x-2+1$

$\boxed{y=-x-1}$

$x\geq-2$

$y=x+2+1$

$\boxed{y=x+3}$

$M = ?$

Gledamo dvije mogućnosti:

1) $x \le -2$

$y = -x-1$

Udaljenost A od točke $T(x,-x-1)$ na $y=-x-1$ je $f(x) = \sqrt{(x-1)^2+(-x-1-0)^2} =$

$= \sqrt{2x^2+2}$. $\quad f'(x) = \frac{4x}{2\sqrt{2x^2+2}} = 0 \Rightarrow \quad x=0$ što nije u domeni

Kako je funkcija f padajuća na $\langle -\infty, 0]$ očito je minimum u $x=-2$ i iznosi

$f(-2) = \sqrt{2\cdot 4+2} = \sqrt{10}$

2) $x \ge -2$

$y = x+3$

$f(x) = \sqrt{(x-1)^2+(x+3)^2} = \sqrt{2x^2+4x+10} \Rightarrow f'(x) = \frac{4x+4}{2\sqrt{2x^2+4x+10}} = 0 \Rightarrow \boxed{x=-1}$

$f(-1) = \sqrt{2-4+10} = \sqrt{8} = 2\sqrt{2}$

$\Rightarrow$ Minimalna udaljenost točke A do $y=|x+1|+1$ je $\underline{2\sqrt{2}}$, i to je udaljenost od $\underline{(-1,2)}$

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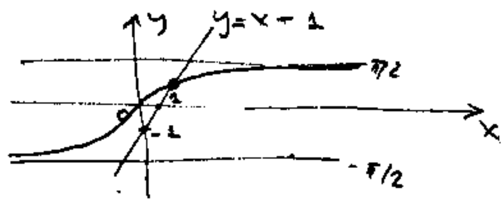
4. Ispitajte tok i nacrtajte graf funkcije $f(x) = \operatorname{arctg} x - x + 1$.

Rješenje. 1) $\mathcal{D}(f) = \mathbb{R}$

2) $N(f) = ?$ $\quad \operatorname{arctg} x = x-1$



Postoji jedna nultočka x_0, $x_0 > 1$.

3) nema vertikalnih asimptota;

horizontalne asimptote: $y = \lim\limits_{x\to-\infty}(\underbrace{\operatorname{arctg} x}_{-\frac{\pi}{2}} - \underbrace{x}_{\infty} + 1)$ nema

$y = \lim\limits_{x\to\infty}(\underbrace{\operatorname{arctg} x}_{\frac{\pi}{2}} - \underbrace{x}_{\infty} + 1)$ nema

kose asimptote: $k = \lim\limits_{x\to\infty} \frac{\operatorname{arctg} x - x + 1}{x} = \lim\limits_{x\to\infty}\left(\frac{\operatorname{arctg} x}{x} - 1 + \frac{1}{x}\right) =$

$= \left(\lim\limits_{x\to\infty} \frac{\operatorname{arctg} x}{x}\right) - 1 + \underbrace{\boxed{\lim\limits_{x\to\infty}\frac{1}{x}}}_{=0} \stackrel{L'H}{=} \lim\limits_{x\to\infty} \frac{\frac{1}{1+x^2}}{1} - 1 + 0 = -1$

0

$l = \lim\limits_{x\to\infty}(\operatorname{arctg} x - \cancel{x} + 1 + \cancel{1\cdot x}) = \frac{\pi}{2}+1$

$\boxed{y = -x + \frac{\pi}{2} + 1}$ je desna kosa asimptota

Za lijevu kosu asimptotu k je isti, a $l = \lim\limits_{x\to-\infty}(\operatorname{arctg} x + 1) = -\frac{\pi}{2}+1 \Rightarrow \underline{y = -x - \frac{\pi}{2} + 1}$

4) $f'(x) = \frac{1}{1+x^2} - 1 = 0 \Rightarrow \quad 1+x^2 = 1 \Rightarrow \underline{x=0}$ kritična točka

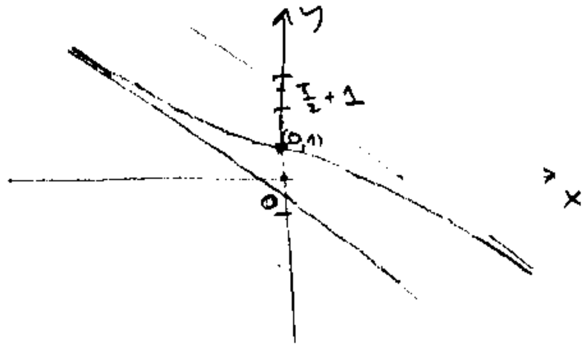
5) $f''(x) = \frac{-1}{(1+x^2)^2}\cdot 2x = 0 \qquad f''(0) = 0 \Rightarrow \quad (f(0) = \operatorname{arctg} 0 - 0 + 1 = 1) \quad \underline{(0,1)}$ točka infleksije

6) tok:

rast $f'(x)>0 \Rightarrow \frac{1}{1+x^2}-1>0 \Rightarrow 1>1+x^2 \Rightarrow x^2<0 \Rightarrow x\in\emptyset$

$\Rightarrow$ funkcija pada na cijelom području definicije

7) graf



5. Bez upotrebe L'Hospitalovog pravila izračunajte $\lim\limits_{x\to 0}\frac{1-4\cos x+3\cos^2 x}{x^2}$.

Rješenje.

$$\lim_{x\to 0}\frac{1-4\cos x+3\cos^2 x}{x^2}=\lim_{x\to 0}\frac{1-4\cos x+3(1-\sin^2 x)}{x^2}=$$

$$=\lim_{x\to 0}\left(\frac{4-4\cos x}{x^2}-3\cdot\left(\frac{\sin x}{x}\right)^2\right)=\lim_{x\to 0}\left(\frac{4(1-\cos x)}{x^2}\right)-3\underbrace{\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2}_{=1}=$$

$$=4\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}-3=8\lim_{x\to 0}\left(\frac{\sin^2\frac{x}{2}}{\frac{x^2}{4}}\cdot\frac{1}{4}\right)-3=2\left(\lim_{\frac{x}{2}\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2-3=$$

koristimo formulu $1-\cos x=2\sin^2\frac{x}{2}$

$$=2-3=\boxed{-1}$$

MATEMATIKA I – 2.7.2002.

RJEŠENJA

1. Odredite domenu funkcije $f(x)=\sqrt{\frac{x^2}{9-x^2}\left(\pi^2-(6\arcsin x)^2\right)}$.

Rješenje. $\frac{x^2}{9-x^2}\cdot\left(\pi^2-(6\arcsin x)^2\right)\geq 0$

$x^2\geq 0 \quad \forall x\in\mathbb{R}$

$\Rightarrow$ ostaje uvjet $\frac{\pi^2-(6\arcsin x)^2}{9-x^2}\geq 0$

1) $\begin{cases}\pi^2-(6\arcsin x)^2\geq 0\\ 9-x^2>0\end{cases}$ $\longrightarrow (\pi-6\arcsin x)(\pi+6\arcsin x)\geq 0$

$9>x^2$

$|x|<3$

a) $\pi-6\arcsin x\geq 0$, $\pi+6\arcsin x\geq 0$

$-\frac{\pi}{6}\leq\arcsin x\leq\frac{\pi}{6}$

$\Rightarrow x\in\left[-\frac{1}{2},\frac{1}{2}\right]$

b) $\pi-6\arcsin x\leq 0$, $\pi+6\arcsin x\leq 0$

$\arcsin x\leq-\frac{\pi}{6}$, $\arcsin x\geq\frac{\pi}{6}$

$\Rightarrow x\in\emptyset$

$\boxed{x\in\left[-\frac{1}{2},\frac{1}{2}\right]}$

2) $\pi^2-(6\arcsin x)^2\leq 0$, $9-x^2<0$ $\longrightarrow$ ovo ni ne treba rješavati, jer je domena arcsin funkcije $[-1,1]$, a zahtjev (*) je $x\in\langle-\infty,3\rangle\cup\langle 3,\infty\rangle$

$9<x^2$

$|x|>3$ (*)

$\Rightarrow \boxed{\mathcal{D}(f)=\left[-\frac{1}{2},\frac{1}{2}\right]}$

---

2. Izračunajte $\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}\cdot\frac{1}{\cos x}-x\operatorname{tg}x\right)$ bez upotrebe L'Hospitalovog pravila.

Rješenje. uvodimo supstituciju $x-\frac{\pi}{2}=t$

$\cos x=\cos\left(\frac{\pi}{2}+t\right)=-\sin t$

$\operatorname{tg}x=\operatorname{tg}\left(\frac{\pi}{2}+t\right)=\frac{\sin\left(\frac{\pi}{2}+t\right)}{\cos\left(\frac{\pi}{2}+t\right)}=\frac{\cos t}{-\sin t}$

Limes postaje $\lim\limits_{t\to 0}\left(-\frac{\pi}{2}\cdot\frac{1}{\sin t}-\left(t+\frac{\pi}{2}\right)\cdot\frac{\cos t}{-\sin t}\right)=$

$=\lim\limits_{t\to 0}\left(-\frac{\pi}{2}\cdot\boxed{\frac{1}{\sin t}}+\frac{t\cos t}{\sin t}+\frac{\pi}{2}\cdot\boxed{\frac{\cos t}{\sin t}}\right)=$

$=\lim\limits_{t\to 0}\left(\frac{\pi}{\cancel{2}}\cdot\frac{1}{\sin t}\underbrace{(\cos t-1)}_{=-\cancel{2}\sin^2\frac{t}{2}}+\frac{t\cos t}{\sin t}\right)=$

$=-\pi\lim\limits_{t\to 0}\frac{\sin^{\cancel{2}}\frac{t}{2}}{2\cancel{\sin\frac{t}{2}}\cdot\cos\frac{t}{2}}+\lim\limits_{t\to 0}\left(\cancel{\frac{t}{\sin t}}^{1}\cancel{\cos t}^{1}\right)=-\pi\lim\limits_{t\to 0}\cancel{\frac{\sin\frac{t}{2}}{\cos\frac{t}{2}}}^{0}+1=\boxed{1}$

3. Nađite kut među krivuljama: $x^2+12x+y^2-4y+15=0$,
$x-y^2+4y-3=0$

Rješenje. Tražimo presjecište dviju krivulja:

$(x+6)^2-36+(y-2)^2-4+15=0 \rightsquigarrow (x+6)^2+(y-2)^2=25$

$x-(y-2)^2+4-3=0 \rightsquigarrow x-(y-2)^2=-1 \;/+$

$x^2+12x+36+x=24$

$x^2+13x+12=0$

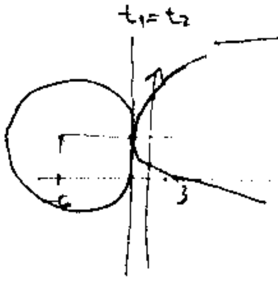
$x_{1,2}=\frac{-13\pm\sqrt{169-48}}{2}=\frac{-13\pm 11}{2}$

$x_1=-12 \Rightarrow (y-2)^2=-12+1$ ↯

$x_2=-1 \Rightarrow \cancel{-1}-(y-2)^2=-\cancel{1}$

$y=2$

$\boxed{(-1,2)}$



- t_1...

$(x+6)^2+(y-2)^2=25$

Implicitno derivamo i dobijemo $y'=-\frac{x+6}{y-2}$

$y'(-1,2)=-\frac{-1+6}{2-2}=\infty=\tan\alpha_1$

$\underbrace{\alpha_1=\frac{\pi}{2}}$

- t_2...

$x-(y-2)^2+1=0$ Implicitno deriviramo i dobijemo

$y'=\frac{1}{2(y-2)} \Rightarrow y'(-1,2)=\frac{-1}{2(2-2)}=\infty=\tan\alpha_2 \Rightarrow \underbrace{\alpha_2=\frac{\pi}{2}}$

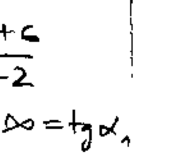
$\Rightarrow$ tangente (obje) su pravci $x_1=x_2=-1$, pa je kut među krivuljama je 0

4. Iz kruga polumjera $r=2$ izrežite kružni isječak koji savijanjem daje stožac maksimalnog obujma.

Rješenje.

$V_S=\frac{\pi x^2}{3}\cdot h$

$I=\frac{r\cdot \ell}{2}$



$2\pi x=\ell$ $\qquad h^2+x^2=4$

$x=\frac{\ell}{2\pi}$ $\qquad x^2=4-h^2$

$\Rightarrow V_S=\frac{\pi}{3}\cdot(4-h^2)\cdot h$

$f(h)=(4-h^2)h=4h-h^3$

$f'(h)=4-3h^2=0 \Rightarrow h^2=\frac{4}{3} \Rightarrow \underbrace{h=\frac{2\sqrt3}{3}}$

$x^2=4-\frac{4}{3}=\frac{8}{3} \Rightarrow$

$\Rightarrow x=\frac{2\sqrt2\cdot\sqrt3}{3}=\frac{2\sqrt6}{3}$

iz $I=\frac{r^2\pi\alpha}{360°}=\frac{r\ell}{2}$ $\Leftarrow$ $\boxed{\ell=\frac{4\pi\sqrt6}{3}}$

$\cancel{2}\cdot\frac{\pi}{360°}\cdot\alpha=\frac{1}{\cancel{2}}\cdot\frac{4\pi}{3}\sqrt6 \Rightarrow \alpha=360°\cdot\frac{\sqrt6}{3}\approx\boxed{294°}$

5. Ispitajte tok i nacrtajte graf funkcije

$f(x)=\frac{3x^2-x}{x+1}$.

Rješenje.

1) $D(f)=\mathbb{R}\setminus\{-1\}$

2) $N(f)=?$

$3x^2-x=0 \Rightarrow x_1=0,\; x_2=1/3$

$N(f)=\{0, 1/3\}$

3) asimptote:

vertikalne $\boxed{x=-1}$

horizontalne $\lim\limits_{x\to\infty} f(x)=\infty$ nema

kosa $k=\lim\limits_{x\to\infty}\frac{f(x)}{x}=\lim\limits_{x\to\infty}\frac{3x^2-x}{x^2+x}\overset{1/x^2}{=}\lim\limits_{x\to\infty}\frac{3-\frac{1}{x}}{1+\frac{1}{x}}=3$

$l=\lim\limits_{x\to\infty}(f(x)-3x)=\lim\limits_{x\to\infty}\left(\frac{\cancel{3x^2}-x-\cancel{3x^2}-3x}{x+1}\right)=\lim\limits_{x\to\infty}\frac{-4x}{x+1}\overset{1/x}{=}\lim\limits_{x\to\infty}\frac{-4}{1+\frac{1}{x}}=-4$

$\Rightarrow \boxed{y=3x-4}$ kosa

4) $f(x)=\frac{3x^2+3x-4x}{x+1}=\frac{3x(x+1)}{x+1}-4\cdot\frac{x}{x+1}=3x-4\cdot\frac{x+1-1}{x+1}=3x-4+\frac{4}{x+1}$

$f'(x)=3-\frac{4}{(x+1)^2}=0 \Rightarrow (x+1)^2=\frac{4}{3}\Rightarrow x+1=\pm\frac{2\sqrt{3}}{3}$

$x_{1/2}=-1\pm\frac{2\sqrt{3}}{3}$ $\quad x_1\approx-2.15$, $x_2\approx 0.15$

$f''(x)=\frac{8}{(x+1)^3}$ $\quad f''(x_1)=\frac{8}{(\cancel{-1}-\frac{2\sqrt{3}}{3}+\cancel{1})^3}<0 \Rightarrow MAX\ (\approx-2.15, \approx-13.92)$

$f''(x_2)=\frac{8}{(-1+\frac{2\sqrt{3}}{3}+1)^3}>0 \Rightarrow MIN\ (\approx 0.15, \approx-0.07)$

$f''(x)=0$ nema točke infleksije

5) rast: $f'(x)>0$

$3>\frac{4}{(x+1)^2}\Rightarrow (x+1)^2>\frac{4}{3}\Rightarrow |x+1|>\frac{2\sqrt{3}}{3}$

$x+1>\frac{2\sqrt{3}}{3}$, $\quad x+1<-\frac{2\sqrt{3}}{3}$

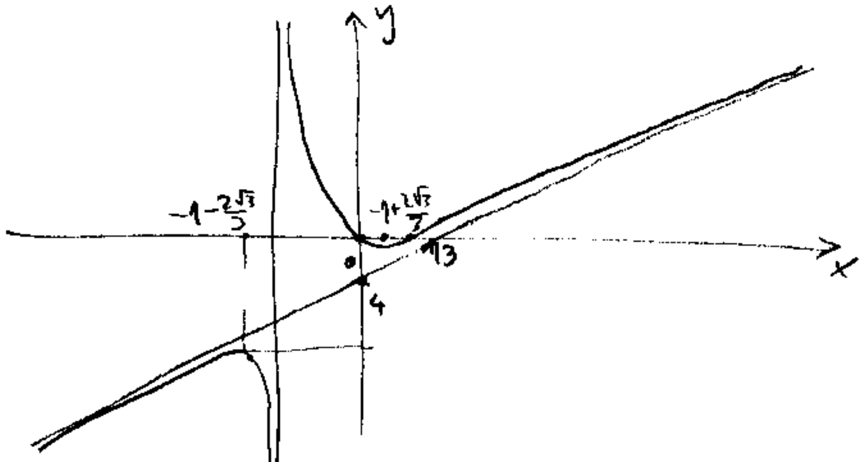
$x>-1+\frac{2\sqrt{3}}{3}$, $\quad x<-1-\frac{2\sqrt{3}}{3}$

	$\langle-\infty,-1-\frac{2\sqrt{3}}{3}\rangle$	$-1-\frac{2\sqrt{3}}{3}$	$\langle-\frac{2\sqrt{3}}{3}-1,2\rangle$	2	$\langle 2,-1+\frac{2\sqrt{3}}{3}\rangle$	$-1+\frac{2\sqrt{3}}{3}$	$\langle-1+\frac{2\sqrt{3}}{3},\infty\rangle$
f'	+	0	−	×	−	0	+
f	↗	MAX	↘	×	↘	MIN	↗

rast: $x\in\langle-\infty,-1-\frac{2\sqrt{3}}{3}\rangle\cup\langle-1+\frac{2\sqrt{3}}{3},\infty\rangle$

pad: $x\in\langle-1-\frac{2\sqrt{3}}{3},-1+\frac{2\sqrt{3}}{3}\rangle\setminus\{-1\}$

6) graf:



1. Odredite $D(f)$, ako je $f(x) = \ln|\ln(15+2x-x^2)|$

Rješenje. • Zbog apsolutne vrijednosti već je zadovoljen uvjet da argument ln funkcije bude pozitivan, osim u slučaju kada je $\ln(15+2x-x^2)=0$

$$15+2x-x^2=1$$
$$x^2-2x-14=0$$
$$x_{1,2} = \frac{2\pm\sqrt{4+56}}{2} = \frac{2\pm 2\sqrt{15}}{2} = \boxed{1\pm\sqrt{15} \Rightarrow \text{ovo treba izbaciti}}$$

• uvjet na drugu logaritamsku funkciju: $15+2x-x^2>0$

$$x^2-2x-15<0$$
$$x_{1,2} = \frac{2\pm\sqrt{4+60}}{2} = \frac{2\pm 8}{2} = 1\pm 4$$
$$x_1=-3,\ x_2=5$$
$$\Rightarrow \boxed{x \in \langle -3,5\rangle}$$

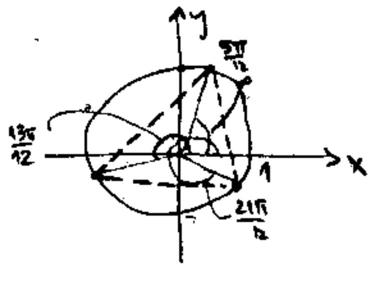
$$\boxed{D(f) = \langle -3,5\rangle \setminus \{1\pm\sqrt{15}\}} \Leftarrow$$

2. Riješite u $\mathbb{C}$: $\sqrt{2}\,x^3+1+i=0$. Slika!

Rješenje.
$$x^3 = \frac{-1-i}{\sqrt2} = -\frac{1}{\sqrt2}-\frac{i}{\sqrt2}$$
$$x_0 = -\frac{\sqrt2}{2} - i\cdot\frac{\sqrt2}{2}$$
$$r=|x_0| = \sqrt{\tfrac12+\tfrac12} = 1$$
$$\cos\varphi = -\frac{\sqrt2}{2},\quad \sin\varphi = -\frac{\sqrt2}{2} \Rightarrow \varphi = \frac{5\pi}{4}$$

$$(x_0)_{0,1,2} = \cos\frac{\varphi+2k\pi}{3} + i\sin\frac{\varphi+2k\pi}{3} \qquad k=0,1,2$$
$$(x_0)_0 = \cos\frac{\frac{5\pi}{4}}{3} + i\sin\frac{5\pi/4}{3} = \boxed{\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}}$$
$$(x_0)_1 = \cos\frac{\frac{5\pi}{4}+2\pi}{3} + i\sin\frac{5\pi/4+2\pi}{3} = \boxed{\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}}$$
$$(x_0)_2 = \cos\frac{5\pi/4+4\pi}{3} + i\sin\frac{5\pi/4+4\pi}{3} = \boxed{\cos\frac{21\pi}{12} + i\sin\frac{21\pi}{12}}$$

Slika:



3. Izračunajte bez upotrebe L'Hospitalovog pravila limes $\lim_{x\to 0}\left(\sin x(\sin x+1)+\cos^2 x\right)^{1/x}$.

Rješenje.

$$\lim_{x\to 0}\left(\sin x(\sin x+1)+\cos^2 x\right)^{1/x} =$$

$$= e^{\ln \lim_{x\to 0}\left(\sin x(\sin x+1)+\cos^2 x\right)^{1/x}} =$$

$$= e^{\lim_{x\to 0} \ln\left(\underbrace{\sin^2 x + \sin x + \cos^2 x}_{=1}\right)^{1/x}} =$$

$$= e^{\lim_{x\to 0}\frac{1}{x}\ln(1+\sin x)} = e^{\lim_{x\to 0}\frac{\ln(1+\sin x)}{\sin x}\cdot\frac{\sin x}{x}} =$$

proširimo sa $\sin x$

$$= e^{\lim_{x\to 0}\frac{\ln(1+\sin x)}{\sin x}\cdot \boxed{\lim_{x\to 0}\frac{\sin x}{x}}} = e^{\lim_{x\to 0}\frac{\ln(1+\sin x)}{\sin x}} =$$

$\boxed{\lim_{x\to 0}\frac{\sin x}{x}}$ "1"

$$= e^{\lim_{\sin x\to 0}\frac{\ln(1+\sin x)}{\sin x}} = e^{\boxed{\lim_{t\to 0}\frac{\ln(1+t)}{t}}} = \boxed{e}$$

$\boxed{\lim_{t\to 0}\frac{\ln(1+t)}{t}}$ "1"

---

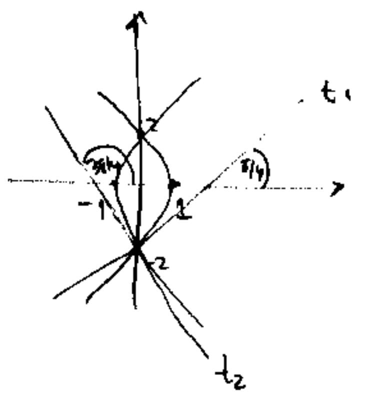
4. Nađite kut između krivulja: $y^2=4-4x$, $y^2=4+4x$. Odredite presječne točke, tangente i nacrtajte sliku.

Rješenje. tražimo presječne točke: $4-4x=4+4x$

$\Rightarrow x=0$

$\Rightarrow y^2=4 \Rightarrow y_{1,2}=\pm 2$

To su točke $\boxed{(0,-2) \text{ i } (0,2)}$



Za $y=-2$ imamo $y_1=-\sqrt{4-4x}$

$y_2=-\sqrt{4+4x}$

$$y_1' = -\frac{1}{2\sqrt{4-4x}}\cdot(-4) = \frac{1}{\sqrt{1-x}}$$

$$y_1'(0) = 1 \Rightarrow \underline{\operatorname{tg}\alpha_1 = 1} \Rightarrow \alpha_1=\frac{\pi}{4}$$

$$y_2' = -\frac{1}{2\sqrt{4+4x}}\cdot 4 = \frac{-1}{\sqrt{1+x}}$$

$$y_2'(0) = -1 \Rightarrow \underline{\operatorname{tg}\alpha_2 = -1} \Rightarrow \alpha_2=-\frac{\pi}{4}$$

$\Rightarrow$ kut između tangenata je $\boxed{\pi/2}$

Postupak za točku $(0,2)$ je analogan.

5. Ispitajte tok i nacrtajte graf funkcije $f(x) = e^{\frac{-1}{x^2}}$.

Rješenje. 1) $\mathcal{D}(f) = \mathbb{R} \setminus \langle 0 \rangle$

2) $N(f) = ?$ nema nultočke $\Rightarrow$ $N(f) = \emptyset$

3) V.A. $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} e^{-\frac{1}{x^2}} = e^{\lim\limits_{x\to 0} -\frac{1}{x^2} \to -\infty} = 0 \Rightarrow$ $\boxed{\lim\limits_{x\to 0} f(x) = 1}$!

H.A. $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} e^{-\frac{1}{x^2} \to 0} = 1$ (vidi graf!)

$\boxed{y = 1}$

4) $f'(x) = e^{-\frac{1}{x^2}} \cdot \frac{2}{x^3} = 0 \Rightarrow$ nema kandidata za ekstrem

5) $f''(x) = e^{-\frac{1}{x^2}} \cdot \frac{2}{x^3} \cdot \frac{2}{x^3} + e^{-\frac{1}{x^2}} \cdot \frac{-6}{x^4} = 0$

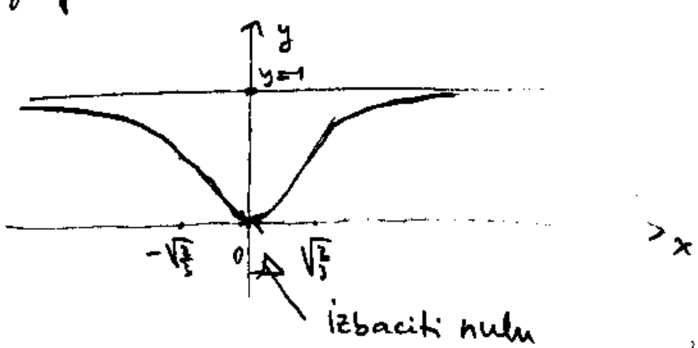
$\frac{2}{x^4} e^{-\frac{1}{x^2}} \left(\frac{2}{x^2} - 3 \right) = 0 \Rightarrow \frac{2}{3} = x^2 \Rightarrow x_{1,2} = \pm\sqrt{\frac{2}{3}} \approx \pm 0{,}81$

6) rast: $f'(x) > 0 \Rightarrow \frac{2}{x^3} > 0 \Rightarrow$ $\boxed{x > 0}$

pad: $\boxed{x < 0}$

	$\langle -\infty, 0 \rangle$	$\langle 0, \infty \rangle$
f'	$-$	$+$
f	$\searrow$	$\nearrow$

7) graf



>> MATEMATIKA I – 11.9.2002. – RJEŠENJA ZADATAKA

1. Riješite u $\mathbb{C}$ jednadžbu: $64z^2 - z^8 = 0$

Rješenje: $z^2(64 - z^6) = 0$

1) $z^6 = 64 = z_0 \quad n=6 \,/\, |z_0| = 64 \,/\, \varphi = 0 \,/\, k = 0 \ldots 5$

$z_k = \sqrt[n]{|z_0|}\left(\cos\frac{\varphi+2k\pi}{n} + i\cdot\sin\frac{\varphi+2k\pi}{n}\right)$

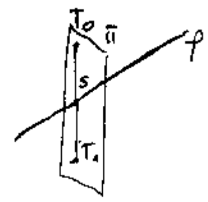
$z_k = \sqrt[6]{64}\left(\cos\frac{0+2k\pi}{6} + i\cdot\sin\frac{0+2k\pi}{6}\right) \Rightarrow z_k = 2\left(\cos\frac{k\pi}{3} + i\sin\frac{k\pi}{3}\right)$

$$\Rightarrow \boxed{z_0 = 2,\ z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 1 + i\sqrt{3},\ z_2 = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = -1 + i\sqrt{3},\ z_3 = -2,\ z_4 = -1 - i\sqrt{3},\ z_5 = 1 - i\sqrt{3}}$$

2) $z^2 = 0 \Rightarrow \boxed{z_6 = z_7 = 0}$

2. Odredite točku simetričnu ishodištu s obzirom na pravac dan jednadžbom $\frac{x-2}{1} = \frac{y}{2} = \frac{z+3}{-3}$.

Rješenje. Ideja: pronađi ravninu π kroz točku $T(0,0,0)$ koja je okomita na zadani pravac p. Presjek $\pi \cap p$ je točka S sa svojstvom da je $2\overrightarrow{T_0S} = \overrightarrow{T_0T_1}$, gdje je T_1 tražena točka.



$p \ldots \frac{x-2}{1} = \frac{y}{2} = \frac{z+3}{-3} \Rightarrow \vec{s}_p = \vec{i} + 2\vec{j} - 3\vec{k}$

$\pi \ldots ?\quad T_0\overset{x_0\,y_0\,z_0}{(0,0,0)},\quad \vec{n}_\pi = \vec{s}_p = \overset{a}{\vec{i}} + \overset{b}{2\vec{j}} \overset{c}{-3\vec{k}}$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0 \Rightarrow x + 2y - 3z = 0 \quad \pi$

$S = \pi \cap p \ldots x = t+2,\ y = 2t,\ z = -3t-3 \Rightarrow$

$t + 2 + 2\cdot 2t - 3(-3t-3) = 0$

$14t = -11 \Rightarrow t = -\frac{11}{14}$

$S = \left(\frac{17}{14}, \frac{-22}{14}, \frac{-9}{14}\right) \Leftarrow \begin{cases} \Rightarrow x_S = -\frac{11}{14} + 2 = \frac{17}{14} \\ y_S = -\frac{22}{14} \\ z_S = \frac{-9}{14} \end{cases}$

$T_1 = (x_1, y_1, z_1) = ?\quad \overrightarrow{T_0T_1} = 2\overrightarrow{T_0S}$

$\Rightarrow (x_1 - 0)\vec{i} + (y_1 - 0)\vec{j} + (z_1 - 0)\vec{k} = 2\left[\left(\frac{17}{14} - 0\right)\vec{i} + \left(\frac{-22}{14} - 0\right)\vec{j} + \left(\frac{-9}{14} - 0\right)\vec{k}\right]$

$\Rightarrow x_1 = \frac{17}{7},\ y_1 = -\frac{22}{7},\ z_1 = \frac{-9}{7} \Rightarrow \boxed{T_1 = \left(\frac{17}{7}, -\frac{22}{7}, \frac{-9}{7}\right)}$

3. Odredite domenu funkcije $f(x) = \log[1 - \log_{3x}(3-x)]$.

Rješenje. uvjeti: $\left.\begin{array}{l} 3x > 0 \Rightarrow x > 0 \\ 3x \neq 1 \Rightarrow x \neq 1/3 \\ 3 - x > 0 \Rightarrow x < 3 \end{array}\right\} \Rightarrow x \in \langle 0, 3\rangle \setminus \{\frac{1}{3}\}$ (*)

$1 - \log_{3x}(3-x) > 0 \ldots$

$\log_{3x}(3-x) < 1$

a) $3x \in \langle 0,1\rangle$

$\boxed{x \in \langle 0, 1/3\rangle}$

$3-x > 3x$

$3 > 4x \Rightarrow \boxed{x < 3/4}$

$\Rightarrow \boxed{x \in \langle 0, \frac{1}{3}\rangle}$

b) $3x \in \langle 1, \infty\rangle$

$\boxed{x \in \langle 1/3, \infty\rangle}$

$3-x < 3x$

$3 < 4x \Rightarrow \boxed{x > 3/4}$

$\Rightarrow \boxed{x \in \langle 3/4, \infty\rangle}$

$\Rightarrow x \in \langle 0, \frac{1}{3}\rangle \cup \langle 3/4, \infty\rangle$, što zajedno s (*) daje

$$\boxed{D(f) = \langle 0, \tfrac{1}{3}\rangle \cup \langle 3/4, 3\rangle}$$

4. Izračunajte bez L'Hospitalovog pravila $\lim\limits_{x\to 1} x^{\frac{x^2-\sqrt{x}}{\sqrt{x}-1}}$.

Rješenje. $$\lim_{x\to 1} x^{\frac{x^2-\sqrt{x}}{\sqrt{x}-1}} = \lim_{x\to 1} e^{\ln x^{\frac{x^2-\sqrt{x}}{\sqrt{x}-1}}} = e^{\lim\limits_{x\to 1} \ln x^{\frac{x^2-\sqrt{x}}{\sqrt{x}-1}}} =$$

$$= e^{\lim\limits_{x\to 1} \frac{x^2-\sqrt{x}}{\sqrt{x}-1}\cdot \ln x} = e^{\lim\limits_{x\to 1} \frac{x^2-\sqrt{x}}{\sqrt{x}-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}\cdot \ln x} =$$

$$\boxed{\lim_{t\to 0} \frac{\ln(t+1)}{t}} = 1$$

$$= e^{\lim\limits_{x\to 1} \frac{(x^2-\sqrt{x})(\sqrt{x}+1)}{x-1}\cdot \ln x} = e^{\lim\limits_{x\to 1} \underbrace{(x^2-\sqrt{x})}_{=0}\cdot\underbrace{(\sqrt{x}+1)}_{2}\cdot \boxed{\lim\limits_{x\to 1}\frac{\ln x}{x-1}}} =$$

$$= e^{0\cdot 2\cdot 1} = e^0 = \boxed{1}$$

5. Odredite kvalitativni graf funkcije $f(x) = (x+1)\cdot \ln^2(x+1)$.

Rješenje 1) $\boxed{D(f) = \langle -1, \infty\rangle}$

2) $N(f) = ?$ $\quad x+1=0 \Rightarrow x=-1$ nije nultočka, jer ne pripada domeni

$\ln^2(x+1)=0 \Rightarrow x+1=1 \Rightarrow x=0 \Rightarrow \boxed{N(f)=\{0\}}$

3) V.A. nema $\left(\lim\limits_{x\to -1_+} f(x) = (\text{2 puta primijeniti L'Hospitalovo pravilo}) = 0\right)$

H.A. $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}(x+1)\cdot\ln^2(x+1) = +\infty$ nema

K.A. $\lim\limits_{x\to+\infty} \frac{f(x)}{x} = \lim\limits_{x\to+\infty} \underbrace{\left(1+\frac{1}{x}\right)}_{1}\cdot\underbrace{\ln^2(x+1)}_{+\infty} = +\infty$ nema

4) $f'(x) = 1\cdot\ln^2(x+1) + \cancel{(x+1)}\cdot 2\ln(x+1)\cdot\frac{1}{\cancel{x+1}} = \ln(x+1)\cdot[\ln(x+1)+2]$

$f'(x)=0 \Rightarrow \ln(x+1)=0 \Rightarrow x+1=1 \Rightarrow \boxed{x=0}$

ili $\ln(x+1)+2=0 \Rightarrow x+1=e^{-2} \Rightarrow \boxed{x=e^{-2}-1}$

stacionarne točke

5) $f''(x) = 2\ln(x+1)\dfrac{1}{x+1} + \dfrac{2}{x+1} = \dfrac{2}{x+1}\left(\ln(x+1)+1\right)$

$f''(0) = 2(1+\ln 1) = 2 > 0 \Rightarrow \quad (0, f(0)) = \boxed{(0,0)\ \text{MIN}}$

$f''(e^{-2}-1) = \dfrac{2}{e^{-2}\cancel{-1}\cancel{+1}}\left(\ln(e^{-2}\cancel{-1}\cancel{+1})+1\right) = 2e^{2}\cdot(-1) = -2e^{2} < 0 \Rightarrow$

$\Rightarrow \quad (e^{-2}-1, f(e^{-2}-1)) = \boxed{(e^{-2}-1, 4e^{-2})\ \text{MAX}}$

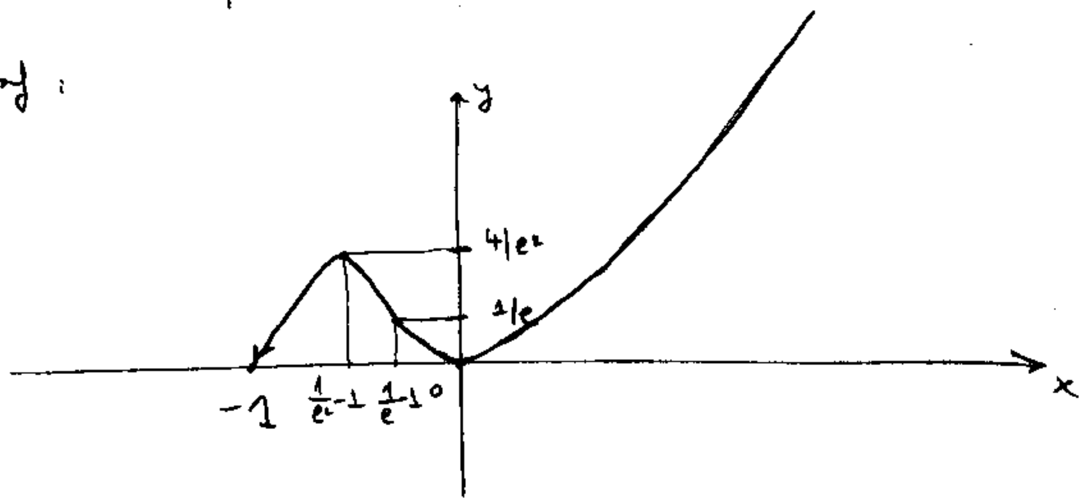
točke infleksije: $f''(x) = 0 \Rightarrow \ln(x+1)+1 = 0 \Rightarrow x+1 = e^{-1} \Rightarrow \underline{x = e^{-1}-1}$

$(e^{-1}-1, f(e^{-1}-1)) = \boxed{(e^{-1}-1, e^{-1})}$ TOČKA INFLEKSIJE

6) rast-pad i tok:

	$\langle -1, e^{-2}-1\rangle$	$e^{-2}-1$	$\langle e^{-2}-1, 0\rangle$	0	$\langle e^{-2}-1, \infty\rangle$
f'	+	0	−	0	+
f	↗	MAX	↘	MIN	↗

7) graf:



MATEMATIKA 1 25.9.2002.

RJEŠENJA ZADATAKA

1. Nađite jednadžbu ravnine koja je okomita na ravninu $\Pi \ldots 2x+y-3z=4$, a sadrži pravac $p \ldots \begin{cases} x-y+z=2 \\ 3x-y+2z=4 \end{cases}$ (*).

Rješenje. Dovoljno je naći jednu točku T i vektor normale $\vec{n}$. Označimo s $\vec{n}_1$ vektor normale ravnine Π. Kako je ravnina koju tražimo okomita na Π, mora biti $\vec{n} \perp \vec{n}_1$.

S druge strane, $\vec{n} \perp \vec{s}$, gdje je $\vec{s}$ vektor smjera pravca p. S obzirom da je p dan kao presjek ravnina, mora biti $\vec{s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 3 & -1 & 2 \end{vmatrix} = -\vec{i}+\vec{j}+2\vec{k}$.

$\vec{n} \perp \vec{n}_1, \vec{n} \perp \vec{s} \Rightarrow \vec{n} = \vec{n}_1 \times \vec{s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -3 \\ -1 & 1 & 2 \end{vmatrix} = \boxed{+5\vec{i}-\vec{j}+3\vec{k}}$

Još treba naći T: možemo uzeti bilo koju točku s pravca p, tj. bilo koju točku koja zadovoljava sustav (*). Uzmimo $\boxed{z=0} \Rightarrow \begin{array}{l} x-y=2 \\ 3x-y=4 \end{array} \Big/ - \Rightarrow 2x=2 \Rightarrow \boxed{x=1}$, $\boxed{y=-1}$

$\Rightarrow T=(1,-1,0)$

$\Rightarrow$ jednadžba ravnine je $5(x-1)-(y+1)+3(z-0)=0$

$$\Rightarrow \boxed{5x-y+3z=6}$$

2. Izračunajte $\lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$ bez upotrebe L'Hospitalovog pravila.

Rješenje.

$$\lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} \cdot \frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}} = \lim_{x\to 0} \frac{\cancel{1}+\sin x-\cancel{1}+\sin x}{x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)} =$$

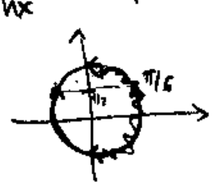
$$= 2 \cdot \underbrace{\boxed{\lim_{x\to 0}\frac{\sin x}{x}}}_{=1} \cdot \boxed{\lim_{x\to 0}\frac{1}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}} = 2\cdot 1\cdot \frac{1}{2} = \boxed{1}$$

$$= \frac{1}{\sqrt{1+\sin 0}+\sqrt{1-\sin 0}} = \frac{1}{2}$$

3. Odredite domenu funkcije $f(x) = \ln \dfrac{2\cos x}{1-2\sin x} + \sqrt{16-x^2}$.

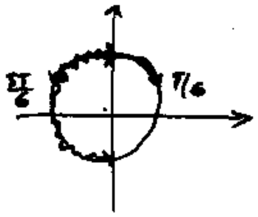
Rješenje. Zbog $\sqrt{\ }$ mora biti $16-x^2 \geq 0 \Rightarrow x^2 \leq 16 \Rightarrow |x| \leq 4$, tj. $\boxed{x \in [-4,4]}$

Zbog ln mora biti $\dfrac{2\cos x}{1-2\sin x} > 0$, tj. imamo dvije mogućnosti:

1) $\begin{array}{l} \cos x > 0 \\ 1-2\sin x > 0 \to \underline{\sin x < 1/2} \end{array}$ 

$\Rightarrow \boxed{x \in \bigcup\limits_{k\in\mathbb{Z}} \langle 2k\pi - \frac{\pi}{2}, 2k\pi + \frac{\pi}{6} \rangle}$

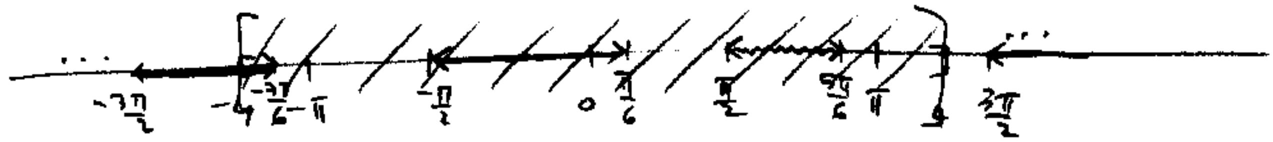
2) $\frac{\cos x < 0}{1-2\sin x < 0} \Rightarrow \sin x > 1/2$



$\Rightarrow \boxed{x \in \bigcup_{k\in\mathbb{Z}} \langle 2k\pi + \frac{\pi}{2}, 2k\pi + \frac{5\pi}{6} \rangle}$

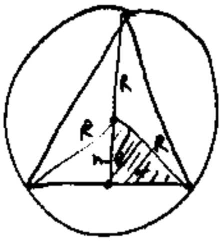
1) ∪ 2): $x \in \left(\bigcup_{k\in\mathbb{Z}} \langle 2k\pi - \frac{\pi}{2}, 2k\pi + \frac{\pi}{6} \rangle\right) \cup \left(\bigcup_{k\in\mathbb{Z}} \langle 2k\pi + \frac{\pi}{2}, 2k\pi + \frac{5\pi}{6} \rangle\right)$

Gledamo presjek ovog skupa sa segmentom $[-4,4]$:



$\Rightarrow \boxed{\mathcal{D}(f) = [-4, -\frac{7\pi}{6}\rangle \cup \langle -\frac{\pi}{2}, \frac{\pi}{6}\rangle \cup \langle \frac{\pi}{2}, \frac{5\pi}{6}\rangle}$

4.



$V = \frac{1}{3} r^2 \pi \cdot h$

$r^2 + (h-R)^2 = R^2 \rightarrow r^2 + h^2 - 2hR = 0 \rightarrow r^2 = 2hR - h^2$

$V = \frac{1}{3}\pi \underbrace{h(2hR - h^2)}_{f(h)}$

$f(h) = 2h^2R - h^3$

$f'(h) = 4hR - 3h^2 = 0 \Rightarrow (h \neq 0) \quad 4R = 3h \Rightarrow \boxed{h = \frac{4}{3}R}$

$r^2 = 2 \cdot \frac{4}{3} R \cdot R - \frac{16}{9} R^2 = \frac{8}{9} R^2 \Rightarrow \quad \boxed{r = \frac{2\sqrt{2}}{3} R}$

$V_{MAX} = \frac{1}{3} \cdot \frac{8}{9} R^2 \cdot \pi \cdot \frac{4}{3} R \Rightarrow \boxed{V_{MAX} = \frac{32}{81} R^3 \pi}$

5. Ispitajte tok i nacrtajte graf funkcije $f(x) = \frac{2x-1}{(x-1)^2}$

Rješenje. 1) $\mathcal{D}(f) = \mathbb{R} \setminus \{1\}$ $N(f) = \{1/2\}$

2) V.A. $x = 1$

H.A. $\lim_{x\to+\infty} f(x) = 0$, isto i za $x \to -\infty \Rightarrow$ $y = 0$ H.A.

K.A. nema

3) $f'(x) = \dots = \frac{-2x}{(x-1)^3} = 0 \Rightarrow \boxed{x = 0}$ stacionarna točka

4) $f''(x) = \dots = \frac{4x+2}{(x-1)^4} = 0 \Rightarrow \boxed{x = -\frac{1}{2}}$ točka infleksije $\boxed{(-\frac{1}{2}, -\frac{8}{9})}$

$f''(0) = 2 > 0 \Rightarrow$ $\boxed{(0, f(0))\text{ lokalni minimum}}$ $\boxed{(0,-1)}$

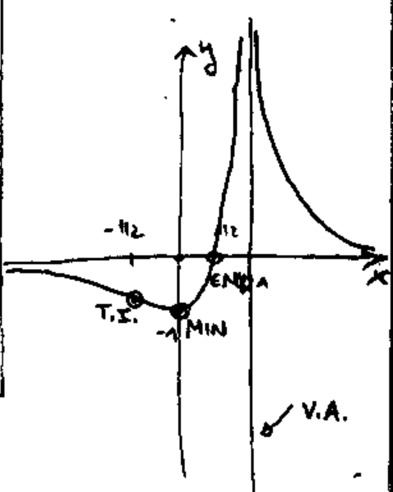
5) rast-pad: $f'(x) > 0 \Rightarrow \frac{x}{(x-1)^3} > 0 \Rightarrow x \in \langle 0,1\rangle$ rast

$\Rightarrow x \in \langle -\infty, 0\rangle \cup \langle 1, \infty\rangle$ pad

6) tt:

	$\langle -\infty, 0\rangle$	0	$\langle 0,1\rangle$	1	$\langle 1, \infty\rangle$
f'	−	0	+		−
f	↘	MIN	↗		↘

$\Rightarrow$ 7) graf:



MATEMATIKA 1 – 1. LISTOPADA 2007. – RJEŠENJA

Zadatak 1. Nađite domenu funkcije $f(x) = \ln(|x|+x) + \arccos\frac{2x-4}{x-1}$.

Rješenje a) $|x|+x>0$ zbog ln

b) $\frac{2x-4}{x-1} \in [-1,1]$ zbog arccos

a) $x \le 0 \Rightarrow -x+x>0$ nema rješenja

$x>0 \Rightarrow x+x>0 \Rightarrow x>0 \Rightarrow \boxed{x \in \langle 0, \infty \rangle}$

b) I) $\frac{2x-4}{x-1} \ge -1 \Rightarrow \frac{2x-4}{x-1}+1 \ge 0 \Rightarrow \frac{2x-4+x-1}{x-1} \ge 0 \Rightarrow \frac{3x-5}{x-1} \ge 0$

kako je $x>0$ (iz a)), onda slijedi $x-1>0 \Rightarrow 3x-5 \ge 0 \Rightarrow \boxed{x \ge \frac{5}{3}}$

II) $\frac{2x-4}{x-1} \le 1 \Rightarrow \frac{2x-4}{x-1}-1 \le 0 \Rightarrow \frac{2x-4-x+1}{x-1} \le 0 \Rightarrow \frac{x-3}{x-1} \le 0$

kako je $x>0$ (iz (a)), slijedi je $x-3 \le 0 \Rightarrow \boxed{x \le 3}$

I) $\cap$ II) daje $x \in [\frac{5}{3}, 3] \Rightarrow \boxed{\mathcal{D}(f) = [\frac{5}{3}, 3]}$

Zadatak 2. Izračunajte $\lim\limits_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$.

Rješenje.
$$\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right) = \lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x} \cdot \frac{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}\right)$$

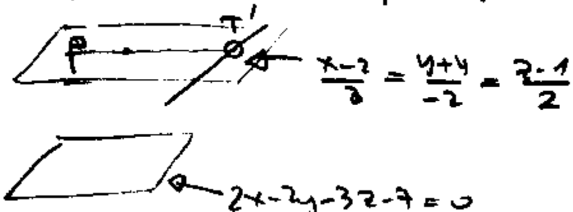
$$= \lim_{x\to\infty} \frac{\cancel{x}+\sqrt{x+\sqrt{x}}-\cancel{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}} \overset{:\sqrt{x}}{=} \lim_{x\to\infty} \frac{\sqrt{1+\frac{\sqrt{x}}{x}}}{\sqrt{1+\frac{\sqrt{x+\sqrt{x}}}{x}}+1} =$$

$$= \lim_{x\to\infty} \frac{\sqrt{1+\underbrace{\frac{1}{\sqrt{x}}}_{\to 0}}}{\sqrt{1+\sqrt{\underbrace{\frac{1}{x}}_{0}+\underbrace{\frac{1}{x\sqrt{x}}}_{0}}}+1} = \boxed{\frac{1}{2}}$$

Zadatak 3.

Odredite jednadžbu pravca koji prolazi točkom $T(3,-2,-4)$, usporedan je ravnini $3x-2y-3z-7=0$ i siječe pravac $\frac{x-2}{3} = \frac{y+4}{-2} = \frac{z-1}{2}$.

Rješenje.



Tražimo točku T' – ona je na presjeku ravnine paralelne sa zadanom (isti vektor normale!), a prolazi točkom T:

$2(x-3)-2(y+2)-3(z+4)=0$

$2x-2y-3z-22=0$

Tražimo presjek te ravnine sa zadanim pravcem: $x=3t+2,\ y=-2t-4,\ z=2t+1$

$2(3t+2)-2(-2t-4)-3(2t+1)-22=0$

$4t=13 \Rightarrow t=\frac{13}{4} \Rightarrow \quad x=\frac{47}{4},\ y=\frac{-21}{2},\ z=\frac{15}{2} \Rightarrow \boxed{T'=\left(\frac{47}{4}, \frac{-21}{2}, \frac{15}{2}\right)}$

Označimo pravac kojeg tražimo s p. Vrijedi $\vec{S}_p=\overrightarrow{TT'}=\frac{35}{4}\vec{i}-\frac{17}{2}\vec{j}+\frac{23}{2}\vec{k}$

Možemo uzeti $\vec{s}_p=35\vec{i}-34\vec{j}+46\vec{k}$, pa je $\boxed{p \ldots \ \frac{x-3}{35}=\frac{y+2}{-34}=\frac{z+4}{46}}$

Zadatak 4. Broj 36 rastavite na dva pozitivna faktora tako da zbroj njihovih kvadrata bude minimalan.

Rješenje. $36=x\cdot y \Rightarrow y=\frac{36}{x}$

$f(x,y)=x^2+y^2$

$\tilde{f}(x)=x^2+\frac{36^2}{x^2}$ ← tražimo minimum ove funkcije

$\tilde{f}'(x)=2x-\frac{36^2}{x^3}\cdot 2=0 \Rightarrow \quad x^4=36^2 \ /\sqrt[4]{\ }$

$x=\sqrt{36}=6$

$y=6$

Radi se o brojevima 6 i 6.

Zadatak 5. Nacrtajte graf funkcije $f(x)=1-e^{-\cos x}$ (funkcija je periodična!).

Rješenje. 1) $\mathcal{D}(f)=\mathbb{R}$

$N(f)=?\quad f(x)=0 \Rightarrow \quad e^{-\cos x}=1=e^0 \Rightarrow \quad \cos x=0 \Rightarrow \boxed{x\in\{k\pi+\frac{\pi}{2} \mid k\in\mathbb{Z}\}}$

$N(f)=\{k\pi+\frac{\pi}{2} \mid k\in\mathbb{Z}\}$

2) Asimptote: nema vertikalnih asimptota

horizontalne: $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\left(1-\frac{1}{e^{\cos x}}\right)$, što ne konvergira, jer funkcija cos ne konvergira ⇒ nema horizontalnih asimptota

Iz istog razloga nema ni kosih asimptota

3) $f'(x)=-e^{-\cos x}\cdot(-\cos x)'=e^{-\cos x}\cdot(-\sin x)=0 \Rightarrow \quad \sin x=0$

$\Rightarrow \boxed{x\in\{k\pi \mid k\in\mathbb{Z}\}}$

stacionarne točke

4) $f''(x)=e^{-\cos x}\cdot(-\cos x)'\cdot(-\sin x)+e^{-\cos x}\cdot(-\sin x)'=$

$=\underbrace{-\sin^2 x}_{\cos^2 x-1}\, e^{-\cos x}+e^{-\cos x}\cdot(-\cos x)=e^{-\cos x}(\cos^2 x-\cos x-1)$

$f''(x)=0 \Rightarrow \cos^2 x-\cos x-1=0 \quad \cos x=t \Rightarrow t^2-t-1=0$

$t_{1,2}=\frac{1\pm\sqrt{1+4}}{2}=\frac{1\pm\sqrt{5}}{2} \Rightarrow \cos x=\frac{1-\sqrt{5}}{2}$ ili $\cos x=\frac{1+\sqrt{5}}{2}>1$ nemoguće

$\boxed{x=\arccos\frac{1-\sqrt{5}}{2}}$ ← to su točke infleksije

Uvrštavamo u f'' stacionarne točke; razlikujemo „parne" i „neparne", tj. one oblika $2l\pi$ i $2l\pi+\pi$, $l\in\mathbb{Z}$:

$f''(2l\pi)=e^{-\cos 2l\pi}(\cos^2 2l\pi-\cos 2l\pi-1)=e^{-1}(\cancel{1}-\cancel{1}-1)=-\frac{1}{e}<0 \Rightarrow$ MAX

$f''(2l\pi+\pi)=e^{-\cos(2l\pi+\pi)}(\cos^2(2l\pi+\pi)-\cos(2l\pi+\pi)-1)=e^{1}(\cancel{1}-(-1)-\cancel{1})=e>0$ MIN

$f(2l\pi)=1-e^{-\overset{1}{\boxed{\cos 2l\pi}}}=1-\frac{1}{e} \Rightarrow$

$f(2l\pi+\pi)=1-e^{-\underset{-1}{\boxed{\cos(2l\pi+\pi)}}}=1-e \Rightarrow$

$(2l\pi,\ 1-\frac{1}{e})$ su točke Maksimuma, $l\in\mathbb{Z}$

$(2l\pi+\pi,\ 1-e)$ su točke minimuma, $l\in\mathbb{Z}$

5) $f'(x)>0 \Rightarrow e^{-\cos x}>0\ \forall x \Rightarrow -\sin x>0 \Rightarrow \sin x<0 \Rightarrow$ $x\in\bigcup_{k\in\mathbb{Z}}\langle 2k\pi-\pi, 2k\pi\rangle$ rast

6) tok: $\Rightarrow$ $x\in\bigcup\langle 2k\pi, 2k\pi+\pi\rangle$ pad

	... -3π		-2π		$-\pi$		0		π		2π	...
f'	... 0	+	0	−	0	+	0	−	0	+	0	...
f	... MIN	↗	MAX	↘	MIN	↗	MAX	↘	MIN	↗	MAX	...

7) graf (funkcija je očito periodična s periodom 2π)