

# MATEMATIKA 1

## PISMENI ISPITI 2003. - RJEŠENJA

15. ožujka

12. travnja

15. studenog

1. Nađite jednadžbu ravnine koja sadrži pravac

$$p_1 \dots \begin{cases} 3x + y - z + 5 = 0 \\ -x + y + z - 3 = 0 \end{cases} \text{ i paralelna je s pravcem}$$

$$p_2 \dots \begin{cases} x + y - z + 5 = 0 \\ x + 3y = 0 \end{cases}$$

2. Odredite domenu funkcije  $f(x) = \log_{\frac{x-5}{7-x}} \left( \frac{\pi}{4} - \arcsin \frac{x}{8} \right)$ .

3. Bez upotrebe L'Hospitalovog pravila izračunajte:  $\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{e^{2x} - 1}$ .

4. U sferu zadanog polumjera  $R$  upišite stožac maksimalnog volumena.

5. Ispitajte tok i nacrtajte graf funkcije  $f(x) = \frac{2x-3}{(x-2)^2}$ .

RJEŠENJA

1. Tražimo ravninu  $\Pi$  koja je određena jednom točkom  $T(x_0, y_0, z_0)$  i vektorom normale  $\vec{n}$ . Kako je  $p_1 \in \Pi$  mora biti  $\vec{n} \perp \vec{s}_1$ , gdje je  $\vec{s}_1$  vektor normale pravca  $p_1$ . Dalje,  $\Pi$  je paralelna s  $p_2$ , pa je  $\vec{n} \perp \vec{s}_2$ , gdje je  $\vec{s}_2$  vektor normale pravca  $p_2$ . To znači da možemo definirati  $\vec{n} = \vec{s}_1 \times \vec{s}_2$ . Računamo  $\vec{s}_1$  i  $\vec{s}_2$ :

$$\vec{s}_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix} = 2\vec{i} - 2\vec{j} + 4\vec{k}, \quad \vec{s}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 1 & 3 & 0 \end{vmatrix} = 3\vec{i} - \vec{j} + 2\vec{k}. \text{ Sada je}$$

jer je  $p_1$  zadan kao presjek dviju ravnina, pa je njegov vektor smjera  $\vec{s}_1$  dan kao vektorski produkt vektora normale tih ravnina.

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & 4 \\ 3 & -1 & 2 \end{vmatrix} = 8\vec{j} + 4\vec{k}$$

Još treba naći točku  $T$ . Kako pravac

$p_1$  pripada  $\Pi$ , možemo ueti bilo koju točku s uprega, tj. bilo koju točku koja zadovoljava

Sustav 
$$\begin{cases} 3x + y - z + 5 = 0 \\ -x + y + z - 3 = 0 \end{cases}$$
. Uzmimo npr.  $x=0$  i zbrojimo jednadžbe  $\Rightarrow$  dobijemo

$$2x + 2y + 2 = 0. \text{ Uz } x=0 \text{ imamo } y=-1 \text{ i uvrštavajući u 1. jednadžbu } z=4.$$

Jednadžba ravnine glasi  $0(x-0) + 8(y+1) + 4(z-4) = 0 \quad /:4$

$$2y + 2 + z - 4 = 0 \Rightarrow \boxed{2y + z - 2 = 0}$$

2. Zbog log funkcije baza  $\frac{x-5}{7-x}$  mora biti veća od nule i različita od 1, a argument

$\frac{\pi}{4} - \arcsin \frac{x}{8}$  veći od nule:

$$1) \frac{x-5}{7-x} > 0 \Rightarrow \begin{cases} a) x-5 > 0 \Rightarrow x > 5 \\ 7-x > 0 \Rightarrow x < 7 \end{cases} \Rightarrow \boxed{x \in (5, 7)} \\ \begin{cases} b) x-5 < 0 \Rightarrow x < 5 \\ 7-x < 0 \Rightarrow x > 7 \end{cases} \Rightarrow \boxed{x \in \emptyset}$$

$$2) \frac{x-5}{7-x} \neq 1 \Rightarrow x-5 \neq 7-x \Rightarrow 2x \neq 12 \Rightarrow \boxed{x \neq 6}$$


$$3) \frac{\pi}{4} - \arcsin \frac{x}{8} > 0 \Rightarrow \arcsin \frac{x}{8} < \frac{\pi}{4} \quad | \sin$$

$$-1 \leq \frac{x}{8} < \frac{\sqrt{2}}{2} \quad | \cdot 8 \Rightarrow -8 \leq x < 4\sqrt{2} \Rightarrow \boxed{x \in [-8, 4\sqrt{2})}$$

$$\mathcal{D}(f) = \boxed{(-8, 4\sqrt{2})}$$

3.  $\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{e^{2x-1} - 1} = \left[ \begin{array}{l} x - \frac{1}{2} = t \\ t \rightarrow 0 \end{array} \middle| \begin{array}{l} 4x^2 - 1 = 4(t + \frac{1}{2})^2 - 1 = \\ = 4(t^2 + t + \frac{1}{4}) - 1 = 4t^2 + 4t \end{array} \right] =$

$= \lim_{t \rightarrow 0} \frac{t^2 + t}{e^{2t} - 1} = \lim_{t \rightarrow 0} \frac{t(t+1)}{\underbrace{(e^{2t}-1)}_1 (t+1)} = \frac{0+1}{e^0+1} = \frac{1}{2}$

4. 
 $(h-R)^2 + r^2 = R^2$   
 $h^2 - 2hR + R^2 + r^2 = R^2$   
 $r^2 = 2hR - h^2$

$V = \frac{1}{3} r^2 \pi h$

$V = \frac{1}{3} \pi h (2hR - h^2)$

$f(h) = 2h^2 R - h^3$

$f'(h) = 4hR - 3h^2 = 0 \Rightarrow h \neq 0$

$4R - 3h = 0$

$h = \frac{4}{3} R$

$r^2 = 2 \cdot \frac{4}{3} R^2 - \frac{16}{9} R^2 = \frac{8}{9} R^2$

$r = \frac{2\sqrt{2}}{3} R$

5.  $f(x) = \frac{2x-3}{(x-2)^2}$

$D(f) = \mathbb{R} \setminus \{2\}$  (2log varijabla)

$N(f) = \left\{ \frac{3}{2} \right\}$  (brujik = 0)

asimptote

V.A.  $x = 2$

H.A.  $\lim_{x \rightarrow -\infty} \frac{2x-3}{(x-2)^2} \stackrel{L'H}{=} \lim_{x \rightarrow -\infty} \frac{2 - \frac{3}{x}}{x - 4 + \frac{4}{x}} = 0_-$  slično  $\lim_{x \rightarrow +\infty} f(x) = 0_+$

K.A. nema

$f'(x) = \frac{2(x-2)^2 - (2x-3) \cdot 2(x-2)}{(x-2)^4} = 2 \cdot \frac{x-2-2x+3}{(x-2)^3} = 2 \cdot \frac{1-x}{(x-2)^3} = 0 \Rightarrow x = 1$

$f''(x) = \frac{2(-1) \cdot (x-2)^3 - 2(1-x) \cdot 3(x-2)^2}{(x-2)^6} = 2 \cdot \frac{-x+2 - (1-x)(x-2)}{(x-2)^4}$

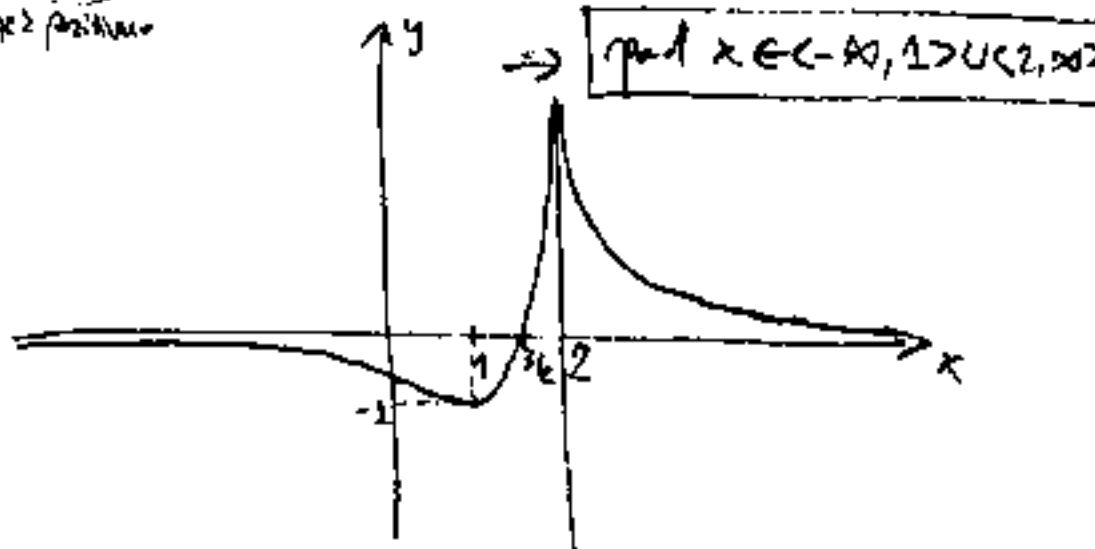
$f''(1) = 2 \cdot \frac{-1+2}{(1-2)^4} > 0 \Rightarrow (1, f(1))$  lokalni minimum  
 $f(1) = \frac{2-3}{(1-2)^2} = -1 \Rightarrow (1, -1)$

rast-pad

$f'(x) > 0 \Rightarrow 2 \cdot \frac{1-x}{(x-2)^3} > 0 \Rightarrow 2 \cdot \frac{1-x}{(x-2)^2(x-2)} > 0 \Rightarrow \frac{1-x}{x-2} > 0 \Rightarrow x \in \langle 1, 2 \rangle$  rast

	$\langle -\infty, 1 \rangle$	$\langle 1, 2 \rangle$	$\langle 2, \infty \rangle$
$f'$	-	+	-
$f$	$\searrow$ MIN	$\nearrow$ MAX	$\searrow$

graf:



1. Riješite sustav koristeći Gaussovu metodu eliminacije:

$$2x + 3y + 4z + 3w = 8$$

$$4x + 2y - z + w = 7$$

$$2x + 5y - 4z = -7$$

$$x - 3y + 4z - 3w = 17$$

2. Odredite tangentu i normalu na graf funkcije  $f(x) = e^{3x-6}$  u točki u kojoj graf siječe pravac  $x - 2 = 0$ .

3. Izračunajte  $\lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{\sin x}$  bez upotrebe L'Hospitalovog pravila.

4. Odredite domenu funkcije  $f(x) = \sqrt{\log_{\frac{1}{2}} x - 3 \log_{\frac{1}{2}} x + 2} + \sqrt{\frac{49x^2 - 1}{1 - 100x^2}}$ .

5. Ispitajte tok i nacrtajte graf funkcije  $f(x) = \frac{(x-1)(x-2)}{x}$ .

1.

$$\begin{bmatrix} 2 & 3 & 4 & 3 & 1 & 8 \\ 4 & 2 & -1 & 1 & 1 & 7 \\ 2 & 5 & -4 & 0 & 1 & -7 \\ 1 & -3 & 4 & -3 & 1 & 17 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 4 & -3 & 1 & 17 \\ 4 & 2 & -1 & 1 & 1 & 7 \\ 2 & 5 & -4 & 0 & 1 & -7 \\ 2 & 3 & 4 & 3 & 1 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 4 & -3 & 1 & 17 \\ 0 & 14 & -17 & 13 & -6 & -61 \\ 0 & 11 & -12 & 6 & -4 & -41 \\ 0 & 9 & -4 & 9 & -2 & -26 \end{bmatrix} \xrightarrow{+} \begin{bmatrix} 1 & -3 & 4 & -3 & 1 & 17 \\ 0 & -8 & 7 & 11 & 2 & 21 \\ 0 & 11 & -12 & 6 & -4 & -41 \\ 0 & 9 & -4 & 9 & -2 & -26 \end{bmatrix} \xrightarrow{+}$$

$$\begin{bmatrix} 1 & -3 & 4 & -3 & 1 & 17 \\ 0 & 1 & 3 & 10 & -5 & -5 \\ 0 & 11 & -12 & 6 & -4 & -41 \\ 0 & 9 & -4 & 9 & -2 & -26 \end{bmatrix} \xrightarrow{-} \begin{bmatrix} 1 & 0 & 13 & 27 & 2 \\ 0 & 1 & 3 & 10 & -5 \\ 0 & 0 & -45 & -104 & 19 \\ 0 & 0 & -32 & -81 & 19 \end{bmatrix} \xrightarrow{\cdot(-11)}$$

$$\begin{bmatrix} 1 & 0 & 13 & 27 & 2 \\ 0 & 1 & 3 & 10 & -5 \\ 0 & 0 & 3645 & 8424 & -1134 \\ 0 & 0 & -3224 & -8424 & 1976 \end{bmatrix} \xrightarrow{\cdot 1} \begin{bmatrix} 1 & 0 & 13 & 27 & 2 \\ 0 & 1 & 3 & 10 & -5 \\ 0 & 0 & 429 & 0 & 1824 \\ 0 & 0 & -3224 & -8424 & 1976 \end{bmatrix} \xrightarrow{\cdot 429}$$

$$\begin{bmatrix} 1 & 0 & 13 & 27 & 2 \\ 0 & 1 & 3 & 10 & -5 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & -3224 & -8424 & 1976 \end{bmatrix} \xrightarrow{\cdot 3} \begin{bmatrix} 1 & 0 & 0 & 27 & -24 \\ 0 & 1 & 0 & 10 & -11 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & -8424 & 18424 \end{bmatrix} \xrightarrow{\cdot(-8424)}$$

$$\begin{bmatrix} 1 & 0 & 0 & 27 & -24 \\ 0 & 1 & 0 & 10 & -11 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{-27} \begin{bmatrix} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \Rightarrow \begin{cases} x=3 \\ y=-1 \\ z=2 \\ w=-1 \end{cases}$$

jedinstveno rješenje

2.  $x-2=0 \Rightarrow x=2 \Rightarrow y=f(2)=e^{3 \cdot 2-6}=e^0=1$

točka u kojoj tražimo tangente i normala je  $(2, 1)$

$$f'(x) = 3 \cdot e^{3x-6}$$

$a_t = f'(2) = 3 \cdot e^0 = 3 \Rightarrow$  tangenta:  $y-1=3(x-2)$

$$\boxed{y = 3x - 5}$$

normala:  $a_n = -\frac{1}{a_t} = -\frac{1}{3} \Rightarrow y-1 = -\frac{1}{3}(x-2)$

$$\boxed{y = -\frac{1}{3}x + \frac{5}{3}}$$

3.  $\lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{\sin x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \lim_{x \rightarrow 0} x \sin \frac{1}{x} = \lim_{x \rightarrow 0} x \sin \frac{1}{x} = \left[ \begin{matrix} x = \frac{1}{t} \\ t \rightarrow 0 \end{matrix} \right] =$

$$= \lim_{t \rightarrow \infty} \frac{1}{t} \sin t = \boxed{0}$$

oscilira u pruzi  $[-1, 1]$

4. Zbog konvencija je  $\log_{\frac{2}{3}} x - 3 \log_{\frac{1}{3}} x + 2 \geq 0$

uvodimo supstituciju  $t = \log_{1/3} x$

$$t^2 - 3t + 2 \geq 0, t_1 = 1, t_2 = 2 \Rightarrow t \leq 1 \text{ ili } t \geq 2$$

$$t \leq 1 \Rightarrow \log_{1/3} x \leq 1 \Rightarrow \boxed{x \geq \frac{1}{3}}$$

$$\text{ili } t \geq 2 \Rightarrow \log_{1/3} x \geq 2 \Rightarrow \boxed{x \leq \frac{1}{9}}$$

Zbog logaritma je  $\boxed{x > 0}$

Zbog konvencija je  $\frac{49x^2-1}{1-100x^2} \geq 0$

a)  $49x^2-1 \geq 0 \Rightarrow x^2 \geq \frac{1}{49} \Rightarrow |x| \geq \frac{1}{7}$   
 $1-100x^2 > 0 \Rightarrow x^2 < \frac{1}{100} \Rightarrow |x| < \frac{1}{10}$

~~.....~~  
 $\frac{1}{10} \quad \frac{1}{7} \quad \frac{1}{10} \quad \frac{1}{7}$

b)  $49x^2-1 < 0 \Rightarrow |x| < \frac{1}{7}$  Nema rješenja  
 $1-100x^2 < 0 \Rightarrow |x| > \frac{1}{10}$

~~.....~~  
 $\frac{1}{10} \quad \frac{1}{7} \quad \frac{1}{10} \quad \frac{1}{7}$

Konačno rješenje:

~~.....~~  
 $\frac{1}{10} \quad 0 \quad \frac{1}{10} \quad \frac{1}{9} \quad \frac{1}{7} \quad \frac{1}{3}$

$$\Rightarrow \boxed{x \in \left[-\frac{1}{7}, -\frac{1}{10}\right) \cup \left(\frac{1}{10}, \frac{1}{9}\right]}$$

$$\boxed{D(f) = \left\langle \frac{1}{10}, \frac{1}{9} \right\rangle}$$

5.  $f(x) = \frac{(x-1)(x-2)}{x^2}$

$D(f) = \mathbb{R} \setminus \{0\}$

$N(f) = \{1, 2\}$

V.A.  $x=0$

H.A.  $\lim_{x \rightarrow \pm\infty} f(x) = \lim_{x \rightarrow \pm\infty} \frac{x^2 - 3x + 2}{x^2} \stackrel{L'H}{=} \lim_{x \rightarrow \pm\infty} \left(1 - \frac{3}{x} + \frac{2}{x^2}\right) = 1 \Rightarrow y=1$  desna i lijeva H.A.

K.A. nema

$f'(x) = \frac{(2x-3)x^2 - (x^2-3x+2) \cdot 2x}{x^4} = \frac{2x^3 - 3x - 2x^3 + 6x - 4}{x^3} = \frac{3x-4}{x^3} = 0 \Rightarrow x = \frac{4}{3}$   
 kandidat za ekstrem

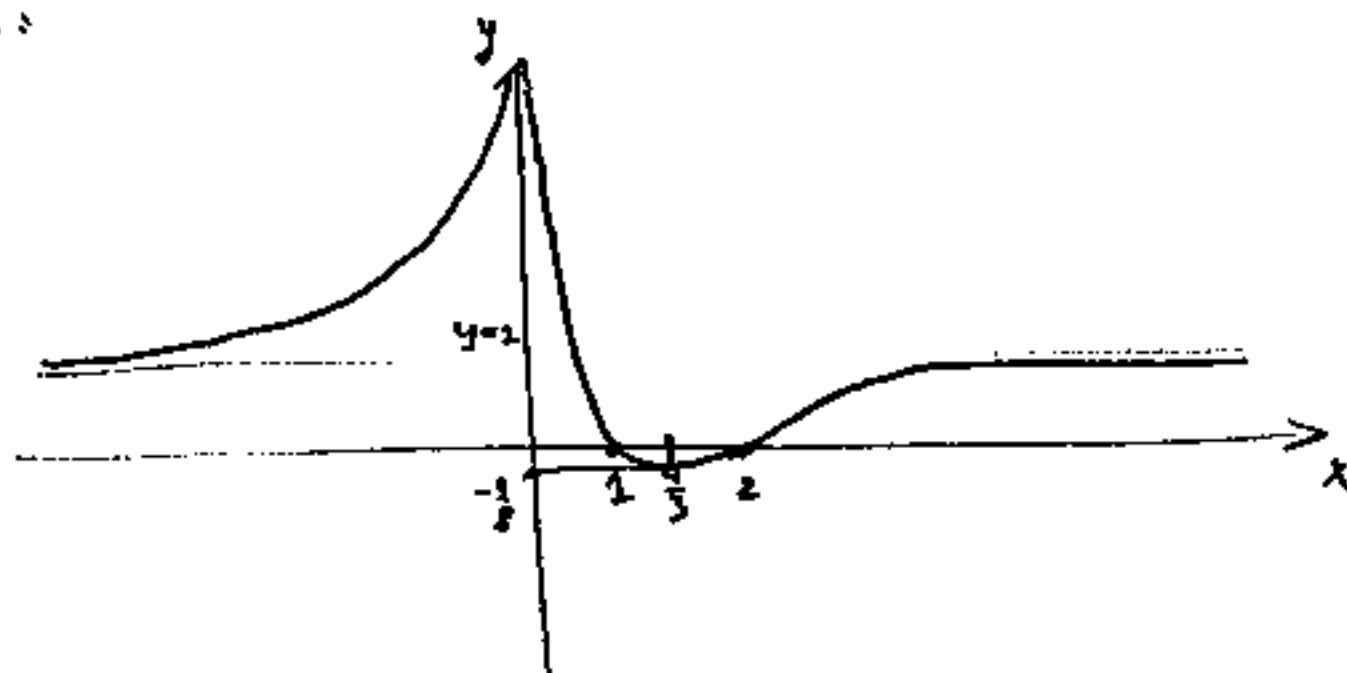
$f''(x) = \frac{3x^2(3x-4) - 3x^2}{x^6} = \frac{3x-9x+12}{x^4} = \frac{12-6x}{x^4} = 0 \Rightarrow x=2$  točka infleksije

$f''\left(\frac{4}{3}\right) = \frac{12 - 6 \cdot \frac{4}{3}}{\left(\frac{4}{3}\right)^4} > 0 \Rightarrow \left(\frac{4}{3}, f\left(\frac{4}{3}\right)\right) = \left(\frac{4}{3}, -\frac{1}{9}\right)$  je lok. minimum

tot.  $f(x) > 0 \Rightarrow \frac{3x-4}{x^3} > 0 \Rightarrow \frac{3x-4}{x^2} > 0 \Rightarrow \frac{3x-4}{x} > 0 \Rightarrow x \in (-\infty, 0) \cup \left(\frac{4}{3}, \infty\right)$  rast  
 $\Rightarrow x \in \left(0, \frac{4}{3}\right)$  pad

	$(-\infty, 0)$	$x=0$	$\left(0, \frac{4}{3}\right)$	$x=\frac{4}{3}$	$(\frac{4}{3}, \infty)$
$f'$	+	*	-	0	+
$f$	$\nearrow$	*	$\searrow$	lok. min	$\nearrow$

graf:



MATEMATIKA 1. - 15.11.2003. - RJEŠENJA

1. Odredite  $x \in \mathbb{R}$  tako da vektori  $\vec{a} = (2x-6)\vec{i} + 4\vec{j} - 3\vec{k}$ ,  $\vec{b} = (3x-1)\vec{i} + 2\vec{j} + 2\vec{k}$ ,  
 $\vec{c} = (3-8x)\vec{i} + (x-2)\vec{j} - 3x\vec{k}$  budu komplanarni.

Rješenje:  $\vec{a}, \vec{b}$  i  $\vec{c}$  će biti komplanarni ako i samo ako zadovoljavaju sledeći  
 uvjet komplanarnosti:

$$\begin{vmatrix} 2x-6 & 4 & -3 \\ 3x-1 & 2 & 2 \\ 3-8x & x-2 & -3x \end{vmatrix} = 0$$

$$\Leftrightarrow (2x-6)(-6x-2x+4) - 4(-3x(3x-1) - 2(3-8x)) - 3 \cdot ((x-2)(3x-1) - 2(3-8x)) = 0$$

$$\Leftrightarrow \text{dobiva se kvadratna jednačina čija su rešenja } \boxed{x_1 = 4} \text{ i } \boxed{x_2 = \frac{+3}{11}}$$

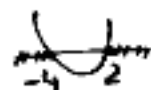
2. Nadite domen funkcije  $f(x) = \frac{\log_{x+7}(x+5)}{\sqrt{x^2+2x-8}}$ .

Rješenje: Imamo sledeće uvjete: 1)  $x+7 > 0$  i  $x+7 \neq 1$  (baza log. funkcije)  
 2)  $x+5 > 0$  (argument log. funkcije)

1)  $\boxed{x > -7}$  i  $\boxed{x \neq -6}$

2)  $\boxed{x > -5}$

3)  $x^2+2x-8 > 0$   
 $x_{1,2} = \frac{-2 \pm \sqrt{36}}{2} = -1 \pm 3 \Rightarrow x_1 = -4$   
 $x_2 = 2$



$\Rightarrow \boxed{x \in \langle -\infty, -4 \rangle \cup \langle 2, \infty \rangle}$

$\rightarrow$  konačno rešenje je:  $\boxed{D(f) = \langle -5, -4 \rangle \cup \langle 2, \infty \rangle}$

3. Na krivulji  $y = x^2 + 2$  nadite tačku najbližu tački  $T(0, 2)$ .

Rješenje: Udaljenost od tačke  $A = (x, y) = (x, x^2 + 2)$  na krivulji od tačke  $T$   
 dana je s  $d(A, T) = \sqrt{(x-0)^2 + (x^2+2)^2} = \sqrt{x^2 + x^4}$ , što  
 shvaćamo kao funkciju po  $x$ . Da bismo našli minimalnu  
 udaljenost, rešavamo  $f'(x) = 0$ , gdje je  $f(x) = \sqrt{x^2 + x^4}$

$$f'(x) = \frac{1}{2\sqrt{x^2+x^4}} \cdot (2x + 4x^3) = 0$$

$$\Rightarrow 2x \cdot (1 + 2x^2) = 0$$

uvjet realiziramo od nule, jer je broj  
 pozitivnog i nenegativnog broja

$$\rightarrow 2x = 0 \Rightarrow \boxed{x_0 = 0} \Rightarrow \boxed{y_0 = 2} \Rightarrow \boxed{A = (0, 2)}$$

4. Izračunajte  $\lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{\sin x}$  bez upotrebe L'Hospitalovog pravila.

Rješenje.  $\lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{\sin x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \boxed{x \sin \frac{1}{x}} = \boxed{0}$   
 1      0      ovdje dio unijet  
 oscilira između -1 i 1

5. Nacrtajte graf funkcije  $f(x) = \frac{\ln x}{x}$ .

Rješenje.

1)  $D(f) = \langle 0, \infty \rangle$  zbog ln funkcije

2)  $N(f) = ?$   $\ln x = 0 \Rightarrow x = 1 \Rightarrow N(f) = \{1\}$

3) v.a.  $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{\ln x}{x} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \rightarrow 0^+} \frac{1}{x} = +\infty$

$\Rightarrow x=0$  je D.V.A.

H.A.  $\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{\ln x}{x} = \lim_{x \rightarrow +\infty} \frac{1}{x} = 0 \Rightarrow y=0$  je D.H.A.

4)  $f'(x) = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2} = 0 \Rightarrow 1 - \ln x = 0$

$\Rightarrow \ln x = 1 \Rightarrow \boxed{x_0 = e}$

$f''(x) = \frac{-\frac{1}{x^2} \cdot x^2 - (1 - \ln x) \cdot 2x}{x^4} = \frac{-x \cdot (1 + (1 - \ln x) \cdot 2)}{x^4}$

$= -\frac{3 - 2 \ln x}{x^3} \Rightarrow f''(e) = -\frac{3 - 2 \ln e}{e^3} = -\frac{1}{e^2} < 0$

$\Rightarrow (e, f(e)) = (e, \frac{\ln e}{e}) = (e, \frac{1}{e})$  lok. max.

točka infleksije:  $f''(x) = 0 \Rightarrow 3 - 2 \ln x = 0 \Rightarrow \ln x = \frac{3}{2} \Rightarrow x = e^{3/2} = \sqrt{e^3}$

$(e^{3/2}, \frac{3}{2} \cdot \frac{1}{e^{3/2}})$  je točka infleksije

5) tč:  $f'(x) > 0 \Rightarrow \frac{1 - \ln x}{x^2} > 0 \Rightarrow 1 - \ln x > 0 \Rightarrow \ln x < 1$   
 $\Rightarrow x < e$

$\Rightarrow x \in \langle 0, e \rangle$  rast  $\Rightarrow x \in \langle e, \infty \rangle$  pad

	$x < 0, e \rangle$	$e$	$\langle e, \infty \rangle$
$f'$	+	0	-
$f$		↗ Max ↘	

