MATEMATIKA 1

PISMENI ISPITI 2003. - RJEŠENJA

15. ožujka
12. travnja
15. studenog

1. Nađite jednadžbu ravnine koja sadrži pravac

$$p_1 \dots \begin{cases} 3x + y - z + 5 = 0 \\ -x + y + z - 3 = 0 \end{cases} \quad \text{i paralelna je s pravcem}$$

$$p_2 \dots \begin{cases} x + y - z + 5 = 0 \\ x + 3y \quad = 0 \end{cases}$$

2. Odredite domenu funkcije $f(x) = \log_{\frac{x-5}{7-x}} \left(\frac{\pi}{4} - \arcsin \frac{x}{8}\right)$.

3. Bez upotrebe L'Hospitalovog pravila izračunajte: $\lim_{x \to \frac{1}{2}} \frac{4x^2-1}{e^{2x-1}-1}$.

4. U sferu zadanog polumjera R upišite stožac maksimalnog volumena.

5. Ispitajte tok i nacrtajte graf funkcije $f(x) = \frac{2x-3}{(x-2)^2}$.

RJEŠENJA

1. Tražimo ravninu Π koja je određena jednom točkom $T(x_0, y_0, z_0)$ i vektorom normale $\vec{n}$. Kako je $p_1 \in \Pi$ mora biti $\vec{n} \perp \vec{s}_1$, gdje je $\vec{s}_1$ vektor normale pravca p_1. Dalje, Π je paralelna s p_2, pa je $\vec{n} \perp \vec{s}_2$, gdje je $\vec{s}_2$ vektor normale pravca p_2. To znači da možemo definirati $\vec{n} = \vec{s}_1 \times \vec{s}_2$. Računamo $\vec{s}_1$ i $\vec{s}_2$:

$$\vec{s}_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix} = 2\vec{i} - 2\vec{j} + 4\vec{k}, \quad \vec{s}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 1 & 3 & 0 \end{vmatrix} = 3\vec{i} - \vec{j} + 2\vec{k}. \text{ Sada je}$$

jer je p_1 zadan kao presjek dviju ravnina, pa je njegov vektor smjera $\vec{s}_1$ dan kao vektorski produkt vektora normala tih ravnina.

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & 4 \\ 3 & -1 & 2 \end{vmatrix} = 8\vec{j} + 4\vec{k}$$

Još treba naći točku T_0. Kako pravac p_1 pripada Π, možemo uzeti bilo koju točku s njega, tj. bilo koju točku koja zadovoljava sustav

$$\begin{aligned} 3x + y - z + 5 &= 0 \\ -x + y + z - 3 &= 0 \end{aligned}$$

. Uzmimo npr. $\underline{x=0}$ i zbrojimo jednadžbe $\Rightarrow$ dobijemo $2x + 2y + 2 = 0$. Uz $x=0$ imamo $\underline{y=-1}$ i uvrstimo u 1. jednadžbu $\underline{z=4}$.

Jednadžba ravnine glasi $\quad 0(x-0) + 8(y+1) + 4(z-4) = 0 \quad /:4$

$$2y + 2 + z - 4 = 0 \Rightarrow \boxed{2y + z - 2 = 0}$$

2. Zbog log funkcije baza $\frac{x-5}{7-x}$ mora biti veća od nule i različita od 1, a argument $\frac{\pi}{4} - \arcsin\frac{x}{8}$ veći od nule:

1) $\frac{x-5}{7-x} > 0 \Rightarrow$ a) $\left.\begin{aligned} x-5>0 &\Rightarrow x>5 \\ 7-x>0 &\Rightarrow x<7 \end{aligned}\right\} \boxed{x \in \langle 5,7\rangle}$

b) $\left.\begin{aligned} x-5<0 &\Rightarrow x<5 \\ 7-x<0 &\Rightarrow x>7 \end{aligned}\right\} \Rightarrow \boxed{x \in \emptyset}$

$\boxed{x \in \langle 5,7\rangle}$

2) $\frac{x-5}{7-x} \neq 1 \Rightarrow x-5 \neq 7-x \Rightarrow 2x \neq 12 \Rightarrow \boxed{x \neq 6}$

3) $\frac{\pi}{4} - \arcsin\frac{x}{8} > 0 \Rightarrow \arcsin\frac{x}{8} < \frac{\pi}{4} \;\Big|\; \sin$

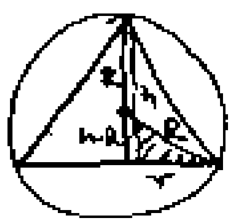
$$-1 \leq \frac{x}{8} < \frac{\sqrt{2}}{2} \;\Big|\cdot 8 \Rightarrow -8 \leq x < 4\sqrt{2} \Rightarrow \boxed{x \in [-8, 4\sqrt{2}\rangle}$$

$\boxed{\mathcal{D}(f) = \langle 5, 4\sqrt{2}\rangle}$

3. $$\lim_{x\to\frac{1}{2}} \frac{4x^2-1}{e^{2x-1}-1} = \left[\begin{array}{c|l} x-\frac{1}{2}=t & 4x^2-1 = 4\left(t+\frac{1}{2}\right)^2-1 = \\ t\to 0 & = 4\left(t^2+t+\frac{1}{4}\right)-1 = 4t^2+4t \end{array}\right] =$$

$$= \lim_{t\to 0} \frac{t^2+t}{e^{2t}-1} = \lim_{t\to 0} \frac{t(t+1)}{\underbrace{(e^t-1)}_{1}(e^t+1)} = \frac{0+1}{e^0+1} = \boxed{\frac{1}{2}}$$

4.



$(h-R)^2+r^2=R^2$

$h^2-2hR+\cancel{R^2}+r^2=\cancel{R^2}$

$\boxed{r^2=2hR-h^2}$

$V=\frac{1}{3}r^2\pi h$

$V=\frac{1}{3}\pi h(2hR-h^2)$

$f(h)=2h^2R-h^3$ $f(h)$

$f'(h)=4hR-3h^2=0 \Rightarrow h\neq 0$

$4R-3h=0$

$\boxed{h=\frac{4}{3}R}$

$r^2=2\cdot\frac{4}{3}R^2-\frac{16}{9}R^2$

$r^2=\frac{8}{9}R^2 \Rightarrow \boxed{r=\frac{2\sqrt{2}}{3}R}$

$V_{max}=\frac{1}{3}\cdot\frac{8}{9}R^2\pi\cdot\frac{4}{3}R \Leftarrow$

$\boxed{V_{MIN}=\frac{32}{81}R^3\pi}$

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5. $f(x)=\dfrac{2x-3}{(x-2)^2}$

$D(f)=\mathbb{R}\setminus\{2\}$ (zbog nazivnika)

$N(f)=\{\frac{3}{2}\}$ (brojnik $=0$)

asimptote

V.A. $x=2$

H.A. $\displaystyle\lim_{x\to-\infty}\frac{2x-3}{\underbrace{(x-2)^2}_{=x^2-4x+4}} \overset{LH}{=} \lim_{x\to-\infty}\frac{2-\cancel{\frac{3}{x}}}{\underbrace{x-4+\cancel{\frac{4}{x}}}_{-\infty}} = 0_-$ slično $\displaystyle\lim_{x\to+\infty}f(x)=0_+$

K.A. nema

$f'(x)=\dfrac{2(x-2)^{\cancel{2}}-(2x-3)\cdot 2\cancel{(x-2)}}{(x-2)^{\cancel{4}}} = 2\cdot\dfrac{x-2-2x+3}{(x-2)^3} = 2\cdot\dfrac{1-x}{(x-2)^3} = 0 \Rightarrow \boxed{x=1}$ kandidat ekstrema

$f''(x)=\dfrac{2(-1)\cdot(x-2)^{\cancel{3}}-2(1-x)\cdot 3\cancel{(x-2)^2}}{(x-2)^{\cancel{6}}} = 2\cdot\dfrac{-x+2-(1-x)(x-2)}{(x-2)^4}$

$f''(1)=2\cdot\dfrac{-1+2}{(1-2)^4}>0 \Rightarrow (1,f(1))$ je $\boxed{\text{lokalni minimum}}$

$f(1)=\dfrac{2-3}{(1-2)^2}=-1 \Rightarrow \boxed{(1,-1)}$

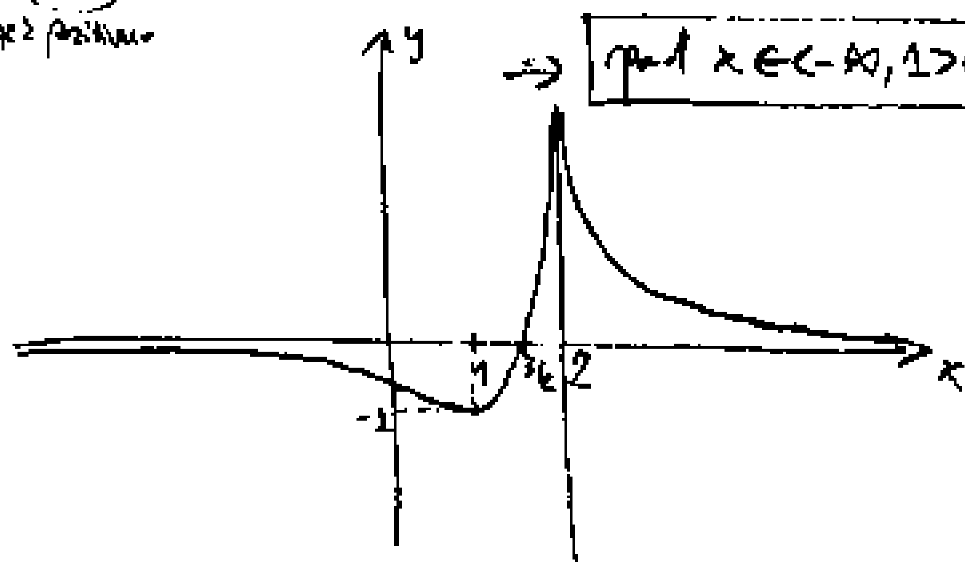
rast-pad

$f'(x)$ rast $\Rightarrow 2\cdot\dfrac{(1-x)}{(x-2)^3}>0 \Rightarrow 2\cdot\dfrac{1-x}{\underbrace{(x-2)^2}_{uvijek\ pozitivno}(x-2)}>0 \Rightarrow \dfrac{1-x}{x-2}>0 \Rightarrow \boxed{x\in\langle 1,2\rangle \text{ rast}}$

$\Rightarrow \boxed{\text{pad } x\in\langle-\infty,1\rangle\cup\langle 2,\infty\rangle}$

	$\langle-\infty,1\rangle$	1	$\langle 1,2\rangle$	2	$\langle 2,\infty\rangle$
f'	$-$	0	$+$	$\times$	$-$
f	$\searrow$	MIN	$\nearrow$	$\times$	$\searrow$

graf: $\longrightarrow$



MATEMATIKA I: 12. travnja 2003.

1. Riješite sustav koristeći Gaussovu metodu eliminacije:

$$\begin{aligned} 2x+3y+4z+3w &= 8 \\ 4x+2y-z+w &= 7 \\ 2x+5y-4z &= -7 \\ x-3y+4z-3w &= 17 \end{aligned}$$

2. Odredite tangentu i normalu na graf funkcije $f(x)=e^{3x-6}$ u točki u kojoj graf siječe pravac $x-2=0$.

3. Izračunajte $\lim_{x\to 0}\frac{x^2\cdot\sin\frac{1}{x}}{\sin x}$ bez upotrebe L'Hospitalovog pravila.

4. Odredite domenu funkcije $f(x)=\sqrt{\log^2_{\frac{1}{2}} x-3\log_{\frac{1}{2}} x+2}+\sqrt{\frac{49x^2-1}{1-100x^2}}$.

5. Ispitajte tok i nacrtajte graf funkcije $f(x)=\frac{(x-1)(x-2)}{e^x}$.

1.

$$\left[\begin{array}{cccc|c} 2 & 3 & 4 & 3 & 8 \\ 4 & 2 & -1 & 1 & 7 \\ 2 & 5 & -4 & 0 & -7 \\ 1 & -3 & 4 & -3 & 17 \end{array}\right] \quad \left[\begin{array}{cccc|c} 1 & -3 & 4 & -3 & 17 \\ 4 & 2 & -1 & 1 & 7 \\ 2 & 5 & -4 & 0 & -7 \\ 2 & 3 & 4 & 3 & 8 \end{array}\right] \begin{array}{c} \\ -4 \\ -2 \\ -2 \end{array} \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & -3 & 4 & -3 & 17 \\ 0 & 14 & -17 & 13 & -61 \\ 0 & 11 & -12 & 6 & -41 \\ 0 & 9 & -4 & 9 & -26 \end{array}\right] (-1)+ \sim \left[\begin{array}{cccc|c} 1 & -3 & 4 & -3 & 17 \\ 0 & -8 & 7 & 4 & 21 \\ 0 & 11 & -12 & 6 & -41 \\ 0 & 9 & -4 & 9 & -26 \end{array}\right] 1+ \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & -3 & 4 & -3 & 17 \\ 0 & 1 & 3 & 10 & -5 \\ 0 & 11 & -12 & 6 & -41 \\ 0 & 9 & -4 & 9 & -26 \end{array}\right] \begin{array}{c} 3 \\ \\ -11 \\ -9 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 13 & 27 & 2 \\ 0 & 1 & 3 & 10 & -5 \\ 0 & 0 & -45 & -104 & 14 \\ 0 & 0 & -31 & -81 & 19 \end{array}\right] \begin{array}{c} \\ \\ /\cdot(-81) \\ /\cdot(104) \end{array} \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 13 & 27 & 2 \\ 0 & 1 & 3 & 10 & -5 \\ 0 & 0 & 3645 & 8424 & -1134 \\ 0 & 0 & -3224 & -8424 & +1976 \end{array}\right] \cdot 1+ \sim \left[\begin{array}{cccc|c} 1 & 0 & 13 & 27 & 2 \\ 0 & 1 & 3 & 10 & -5 \\ 0 & 0 & 421 & 0 & 824 \\ 0 & 0 & -3224 & -8424 & 1976 \end{array}\right] /:421 \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 13 & 27 & 2 \\ 0 & 1 & 3 & 10 & -5 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & -3224 & -8424 & 1976 \end{array}\right] \begin{array}{c} -13 \\ -3 \\ \\ 3224 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 27 & -24 \\ 0 & 1 & 0 & 10 & -11 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & -8424 & 8424 \end{array}\right] /:(-8424)$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 27 & -24 \\ 0 & 1 & 0 & 10 & -11 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \begin{array}{c} -27 \\ -10 \\ \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \Rightarrow \boxed{\begin{array}{c} x=3 \\ y=-1 \\ z=2 \\ w=-1 \end{array}}$$

jedinstveno rješenje

2. $x-2=0 \Rightarrow x=2 \Rightarrow y=f(2)=e^{3\cdot 2-6}=e^0=1$

točka u kojoj tražimo tangentu i normalu je $(2,1)$

$f'(x)=3\cdot e^{3x-6}$

$a_t=f'(2)=3\cdot e^0=3 \Rightarrow$ tangenta: $y-1=3(x-2)$

$$\boxed{y=3x-5}$$

normala: $a_n=-\frac{1}{a_t}=-\frac{1}{3} \Rightarrow y-1=-\frac{1}{3}(x-2)$

$$\boxed{y=-\frac{1}{3}x+\frac{5}{3}}$$

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3. $$\lim_{x\to 0}\frac{x^2\sin\frac{1}{x}}{\sin x}=\underbrace{\boxed{\lim_{x\to 0}\frac{x}{\sin x}}}_{=1}\cdot\lim_{x\to 0}x\sin\frac{1}{x}=\lim_{x\to 0}x\sin\frac{1}{x}=\begin{bmatrix}x=\frac{1}{t}\\ t\to\infty\end{bmatrix}=$$

$$=\boxed{\lim_{t\to\infty}\underbrace{\frac{1}{t}}_{0}\underbrace{\sin t}}=\boxed{0}$$

oscilira u granici $[-1,1]$

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4. Zbog korijena je $\log^2_{\frac{1}{3}}x-3\log_{\frac{1}{3}}x+2\geq 0$

uvodimo supstituciju $t=\log_{1/3}x$

$t^2-3t+2\geq 0$, $t_1=1, t_2=2 \Rightarrow t\leq 1$ ili $t\geq 2$

$t\leq 1 \Rightarrow \log_{1/3}x\leq 1 \Rightarrow \boxed{x\geq\frac{1}{3}}$ ili

ili $t\geq 2 \Rightarrow \log_{1/3}x\geq 2 \Rightarrow \boxed{x\leq\frac{1}{9}}$

Zbog logaritma je $\boxed{x>0}$

Zbog korijena je $\frac{49x^2-1}{\underbrace{1-100x^2}_{\neq 0}}\geq 0$

a) $49x^2-1\geq 0 \Rightarrow x^2\geq\frac{1}{49}\ /\sqrt{\ } \Rightarrow |x|\geq\frac{1}{7}$

$1-100x^2>0 \Rightarrow x^2<\frac{1}{100}\ /\sqrt{\ } \Rightarrow |x|<\frac{1}{10}$

$-\frac{1}{7}\quad -\frac{1}{10}\quad \frac{1}{10}\quad \frac{1}{7}$

Nema rješenja

b) $49x^2-1\leq 0 \Rightarrow |x|\leq\frac{1}{7}$

$1-100x^2<0 \Rightarrow |x|>\frac{1}{10}$

$-\frac{1}{7}\quad -\frac{1}{10}\quad \frac{1}{10}\quad \frac{1}{7}$

$\Rightarrow \boxed{x\in\left[-\frac{1}{7},-\frac{1}{10}\right\rangle\cup\left\langle\frac{1}{10},\frac{1}{7}\right]}$

Konačno rješenje:

$-\frac{1}{7}\quad -\frac{1}{10}\quad 0\quad \frac{1}{10}\quad \frac{1}{9}\quad \frac{1}{7}\quad \frac{1}{3}$

$$\boxed{\mathcal{D}(f)=\left\langle\frac{1}{10},\frac{1}{9}\right]}$$

5. $f(x) = \frac{(x-1)(x-2)}{x^2}$

$D(f) = \mathbb{R} \setminus \{0\}$

$N(f) = \{1, 2\}$

V.A. $x = 0$

H.A. $\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x^2-3x+2}{x^2} \Big| :x^2 = \lim_{x \to \pm\infty} \left(1 - \boxed{\cancel{\frac{3}{x}} + \cancel{\frac{2}{x^2}}}^{\,0}\right) = 1 \Rightarrow$ $y=1$ desna i lijeva H.A.

K.A. nema

$f'(x) = \frac{(2x-3)x^{\cancel{2}} - (x^2-3x+2)\cdot 2\cancel{x}}{x^{\cancel{4}}} = \frac{\cancel{2x^2}-3x-\cancel{2x^2}+6x-4}{x^3} = \frac{3x-4}{x^3} = 0 \Rightarrow$ $x = \frac{4}{3}$ kandidat za ekstrem

$f''(x) = \frac{3x^{\cancel{3}} - (3x-4)\cdot 3x^{\cancel{2}}}{x^{\cancel{6}}} = \frac{3x-9x+12}{x^4} = \frac{12-6x}{x^4} = 0 \Rightarrow$ $x = 2$ točka infleksije

$f''\left(\frac{4}{3}\right) = \frac{12 - 6^2 \cdot \frac{4}{3}}{\left(\frac{4}{3}\right)^4_{>0}} > 0 \Rightarrow$ $\left(\frac{4}{3}, f\left(\frac{4}{3}\right)\right) = \left(\frac{4}{3}, -\frac{1}{8}\right)$ je lok. minimum

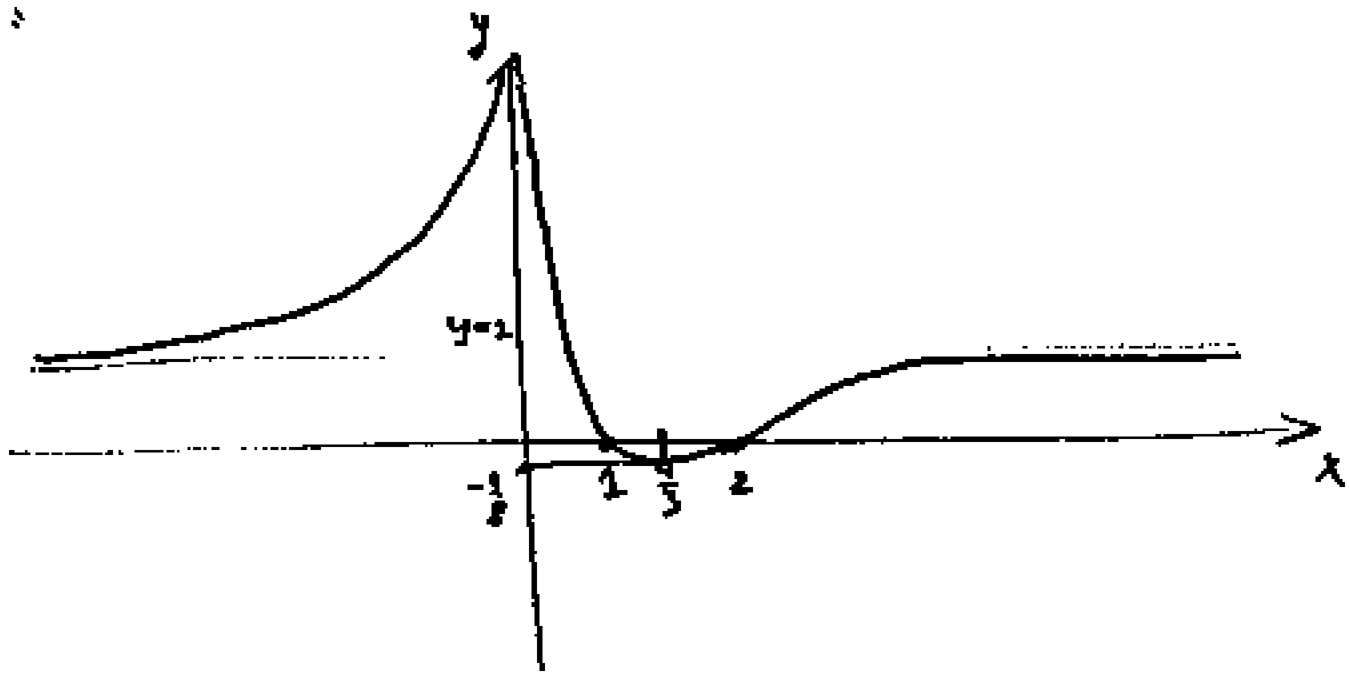
tok: $f'(x) > 0 \Rightarrow \frac{3x-4}{x^3} > 0 \Rightarrow \frac{3x-4}{x^2 x} > 0 \Rightarrow \frac{3x-4}{x} > 0 \Rightarrow x \in \langle -\infty, 0\rangle \cup \langle \frac{4}{3}, \infty\rangle$ rast

$x^2 > 0$ uvijek

$\Rightarrow x \in \langle 0, \frac{4}{3}\rangle$ pad

	$\langle -\infty, 0\rangle$	$x=0$	$\langle 0, \frac{4}{3}\rangle$	$x=4/3$	$\langle 4/3, \infty\rangle$
f'	+	×	−	0	+
f	↗	×	↘	MIN	↗

graf:



MATEMATIKA 1. – 15.11.2003. – RJEŠENJA

1. Odredite $x \in \mathbb{R}$ tako da vektori $\vec{a}=(2x-6)\vec{i}+4\vec{j}-3\vec{k}$, $\vec{b}=(3x-1)\vec{i}+2\vec{j}+2\vec{k}$, $\vec{c}=(3-8x)\vec{i}+(x-2)\vec{j}-3x\vec{k}$ budu komplanarni.

<u>Rješenje:</u> $\vec{a}, \vec{b}$ i $\vec{c}$ će biti komplanarni ako i samo ako zadovoljavaju sljedeći uvjet komplanarnosti:

$$\begin{vmatrix} 2x-6 & 4 & -3 \\ 3x-1 & 2 & 2 \\ 3-8x & x-2 & -3x \end{vmatrix} = 0$$

$\Leftrightarrow \quad (2x-6)(-6x-2x+4)-4(-3x(3x-1)-2(3-8x))-3\cdot$

$\cdot((x-2)(3x-1)-2(3-8x))=0$

$\Leftrightarrow$ dobiva se kvadratna jednadžba čija su rješenja $\boxed{x_1=4}$ i $\boxed{x_2=\frac{+3}{11}}$.

2. Nađite domenu funkcije $f(x)=\dfrac{\log_{x+7}(x+5)}{\sqrt{x^2+2x-8}}$.

<u>Rješenje.</u> Imamo sljedeće uvjete:
1) $x+7>0$ i $x+7\neq 1$ (baza log. funkcije)
2) $x+5>0$ (argument log. funkcije)
3) $x^2+2x-8>0$ (funkcija $f(x)=\sqrt{x}$ – mora biti stroga nejednakost jer za jednakost imamo nulu u nazivniku)

1) $\boxed{x>-7}$ i $\boxed{x\neq -6}$

2) $\boxed{x>-5}$

3) $x^2+2x-8=0$

$x_{1,2}=\frac{-2\pm\sqrt{36}}{2}=-1\pm 3 \Rightarrow x_1=-4$, $x_2=2$

$\Rightarrow \boxed{x\in\langle-\infty,-4\rangle\cup\langle 2,\infty\rangle}$

$\rightarrow$ konačno rješenje je: $\boxed{\mathcal{D}(f)=\langle-5,-4\rangle\cup\langle 2,\infty\rangle}$

3. Na krivulji $y=x^2+2$ nađite točku najbližu točki $T(0,2)$.

<u>Rješenje</u> Udaljenost d točke $A=(x,y)=(x,x^2+2)$ na krivulji od točke T dana je s $d(A,T)=\sqrt{(x-0)^2+(x^2+\cancel{2}-\cancel{2})^2}=\sqrt{x^2+x^4}$, što shvaćamo kao funkciju po x. Da bismo našli minimalnu udaljenost, rješavamo $f'(x)=0$, gdje je $f(x)=\sqrt{x^2+x^4}$

$$f'(x)=\frac{1}{2\sqrt{x^2+x^4}}\cdot(2x+4x^3)=0$$

$$\Rightarrow \quad 2x\cdot\underbrace{(1+2x^2)}_{\text{uvjet različito od nule, jer je zbroj pozitivnog i nenegativnog broja}}=0$$

$$\rightarrow \quad 2x=0 \Rightarrow \boxed{x_0=0} \Rightarrow \boxed{y_0=2} \Rightarrow \boxed{A=(0,2)}$$

4. Izračunajte $\lim\limits_{x\to 0} \frac{x^2\sin\frac{1}{x}}{\sin x}$ bez upotrebe L'Hospitalovog pravila.

Rješenje. $$\lim_{x\to 0} \frac{x^2\sin\frac{1}{x}}{\sin x} = \lim_{x\to 0} \underset{1}{\underbrace{\frac{x}{\sin x}}}\cdot \underset{0}{\underbrace{x}}\,\underset{\text{ovaj dio uvijek oscilira između -1 i 1}}{\boxed{\sin\frac{1}{x}}} = \boxed{0}$$

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5. Nacrtajte graf funkcije $f(x)=\frac{\ln x}{x}$.

Rješenje.

1) $\mathcal{D}(f)=\langle 0,\infty\rangle$ zbog ln funkcije

2) $\mathcal{N}(f)=?$ $\quad \ln x=0 \Rightarrow x=1 \Rightarrow \mathcal{N}(f)=\{1\}$

3) V.A. $\lim\limits_{x\to 0+} f(x)=\lim\limits_{x\to 0+}\frac{\ln x}{x}=\lim\limits_{x\to 0+}\frac{\frac{1}{x}}{1}=\lim\limits_{x\to 0+}\frac{1}{x}=+\infty$

$\Rightarrow$ $x=0$ je D.V.A.

H.A. $\lim\limits_{x\to +\infty} f(x)=\lim\limits_{x\to+\infty}\frac{\ln x}{x}=\lim\limits_{x\to+\infty}\frac{1}{x}=0 \Rightarrow y=0$ je D.H.A.

4) $f'(x)=\frac{\frac{1}{x}\cdot x-\ln x\cdot 1}{x^2}=\frac{1-\ln x}{x^2}=0\Rightarrow 1-\ln x=0\Rightarrow$

$\Rightarrow \ln x=1\Rightarrow \boxed{x_0=e}$

$f''(x)=\frac{-\frac{1}{\cancel{x}}\cdot x^{\cancel{2}}-(1-\ln x)\cdot 2x}{x^4}=\frac{-\cancel{x}\cdot(1+(1-\ln x)\cdot 2)}{x^{\cancel{4}}}=$

$=-\frac{3-2\ln x}{x^3}\Rightarrow f''(e)=-\frac{3-2\ln e}{e^3}=-\frac{1}{e^3}<0$

$\Rightarrow (e,f(e))=(e,\frac{\ln e}{e})=\boxed{(e,\frac{1}{e})\text{ lok. max}}$

točka infleksije: $f''(x)=0\Rightarrow 3-2\ln x=0\Rightarrow \ln x=\frac{3}{2}\Rightarrow x=e^{3/2}=\sqrt{e^3}$

$(e^{3/2},\frac{3}{2}\cdot\frac{1}{e^{3/2}})$ je točka infleksije

5) r+p: $f'(x)>0\Rightarrow \frac{1-\ln x}{\underset{>0\text{ uvijek}}{x^2}}>0\Rightarrow 1-\ln x>0\Rightarrow \ln x<1$

$\Rightarrow x<e$

$\Rightarrow x\in\langle 0,e\rangle$ rast $\Rightarrow x\in\langle e,\infty\rangle$ pad

	0	$\langle 0,e\rangle$	e	$\langle e,\infty\rangle$
f'	X	+	0	−
f	X	↗	max	↘

6) graf: